Chapter 6

Automata Theory

Instructions

A. The following abbreviations are used in this chapter.

```
FSM    -  Finite State Machine
DFSM   -  Deterministic Finite State Machine
NDFSM  -  Non-Deterministic Finite State Machine
PDM    -  Push Down Machine
DPDM   -  Deterministic Push Down Machine
NDPDM  -  Non-Deterministic Push Down Machine
TM     -  Turing Machine
UTM    -  Universal Turing Machine
CFG    -  Context Free Grammar
CF     -  Context Free
CFL    -  Context Free Language
CSG    -  Context Sensitive Grammar
```

B. In Transition diagrams, states are represented by circles.
The `start` state is represented by a circle pointed to by an arrow.
A `final` state is represented by a circle encircled by another.

C. In a CFG, unless stated otherwise, grammar symbol on the left hand side of the first production, is the `start` symbol.

1. The word 'formal' in formal languages means
(a) the symbols used have well-defined meaning
(b) they are unnecessary, in reality
(c) only the form of the string of symbols is significant
(d) none of the above

2. Let $A = \{0, 1\}$. The number of possible strings of length 'n' that can be formed by the elements of the set A is

(a) $n!$ (b) n^2 (c) n^n (d) 2^n

3. Choose the correct statements.

(a) Moore and Mealy machines are FSM's with output capability.
(b) Any given Moore machine has an equivalent Mealy machine.
(c) Any given Mealy machine has an equivalent Moore machine.
(d) Moore machine is not an FSM.

4. The major difference between a Moore and a Mealy machine is that

(a) the output of the former depends on the present state and the current input
(b) the output of the former depends only on the present state
(c) the output of the former depends only on the current input
(d) none of the above

5. Choose the correct statements.

(a) A Mealy machine generates no language as such.
(b) A Moore machine generates no language as such.
(c) A Mealy machine has no terminal state.
(d) For a given input string, length of the output string generated by a Moore machine is one more than the length of the output string generated by that of a Mealy machine.

***6.** The recognizing capability of NDFSM and DFSM

(a) may be different (b) must be different
(c) must be the same (d) none of the above

7. FSM can recognize

(a) any grammar (b) only CFG
(c) any unambiguous grammar (d) only regular grammar

8. Pumping lemma is generally used for proving

(a) a given grammar is regular
(b) a given grammar is not regular
(c) whether two given regular expressions are equivalent
(d) none of the above

***9.** Which of the following are not regular?

(a) String of 0's whose length is a perfect square.
(b) Set of all palindromes made up of 0's and 1's.
(c) Strings of 0's, whose length is a prime number.
(d) String of odd number of zeroes.

***10.** Which of the following pairs of regular expressions are equivalent?

(a) `1(01)*` and `(10)*1` (b) `x(xx)*` and `(xx)*x`
(c) `(ab)*` and `a*b*` (d) x^+ and x^*x^+

11. Choose the correct statements.

(a) A = {$a^n b^n$ | n=0,1,2,3,...} is a regular language.

(b) The set B, consisting of all strings made up of only a's and b's having equal number of a's and b's defines a regular language.

(c) *L*(A*B*) ∩ B gives the set A.

(d) None of the above

***12.** Pick the correct statements.

The logic of Pumping lemma is a good example of

(a) the Pigeon-hole principle
(b) the divide and conquer technique
(c) recursion
(d) iteration

***13.** The basic limitation of an FSM is that

(a) it can't remember arbitrary large amount of information
(b) it sometimes recognizes grammars that are not regular
(c) it sometimes fails to recognize grammars that are regular
(d) all of the above

14. Palindromes can't be recognized by any FSM because

(a) an FSM can't remember arbitrarily large amount of information
(b) an FSM can't deterministically fix the mid-point
(c) even if the mid-point is known, an FSM can't find whether the second half of the string matches the first half
(d) none of the above

15. An FSM can be considered a TM

(a) of finite tape length, rewinding capability and unidirectional tape movement
(b) of finite tape length, without rewinding capability and unidirectional tape movement
(c) of finite tape length, without rewinding capability and bidirectional tape movement
(d) of finite tape length, rewinding capability and bidirectional tape movement

16. TM is more powerful than FSM because

(a) the tape movement is confined to one direction
(b) it has no finite state control
(c) it has the capability to remember arbitrary long sequences of input symbols.
(d) none of the above

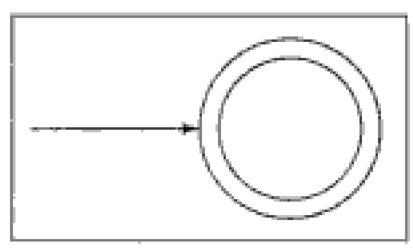

Fig. 6.1

***17.** The FSM pictured in Fig. 6.1 recognizes

(a) all strings
(b) no string
(c) ε – alone
(d) none of the above

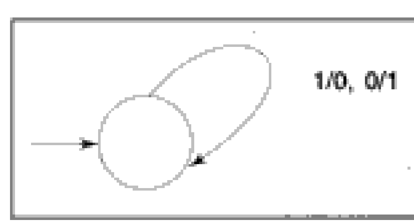

Fig. 6.2

18. The FSM pictured in Fig. 6.2 is a

(a) Mealy machine
(b) Moore machine
(c) Kleene machine
(d) none of the above

19. The above machine

(a) complements a given bit pattern
(b) generates all strings of 0's and 1's
(c) adds 1 to a given bit pattern
(d) none of the above

20. The language of all words (made up of a's and b's) with at least two a's can be described by the regular expression

(a) `(a+b)*a(a+b)*a(a+b)*`
(b) `(a+b)*ab*a(a+b)*`
(c) `b*ab*a(a+b)*`
(d) `a(a+b)*a(a+b)*(a+b)*`

21. Which of the following pairs of regular expression are not equivalent?

(a) `(ab)*a and a(ba)*`
(b) `(a+b)* and (a*+b)*`
(c) `(a*+b)* and (a+b)*`
(d) none of the above

***22.** Consider the two FSM's in Fig. 6.3.
Pick the correct statement.

(a) Both are equivalent
(b) The second FSM accepts only ε
(c) The first FSM accepts nothing
(d) None of the above

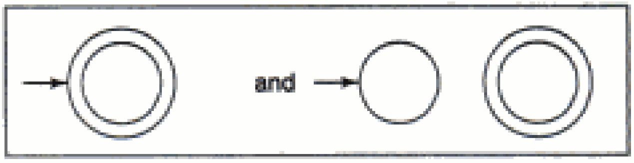

Fig. 6.3

23. Set of regular languages over a given alphabet set, is not closed under

(a) union
(b) complementation
(c) intersection
(d) none of the above

***24.** The machine pictured in Fig. 6.4.

(a) complements a given bit pattern
(b) finds 2's complement of a given bit pattern
(c) increments a given bit pattern by 1
(d) changes the sign bit

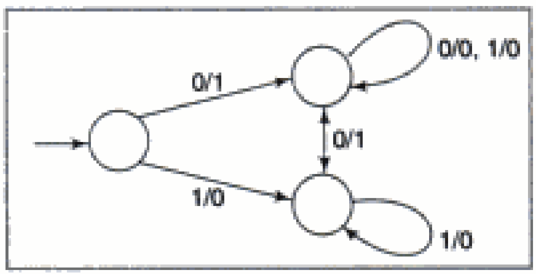

Fig. 6.4

25. For which of the following applications regular expressions can't be used?

(a) Designing compilers
(b) Developing text editors
(c) Simulating sequential circuits
(d) Designing computers

***26.** The FSM pictured in Fig. 6.5 recognizes

(a) any string of odd number of a's
(b) any string of odd number of a's and even number of b's
(c) any string of even number of a's and even number of b's
(d) any string of even number of a's and odd number of b's

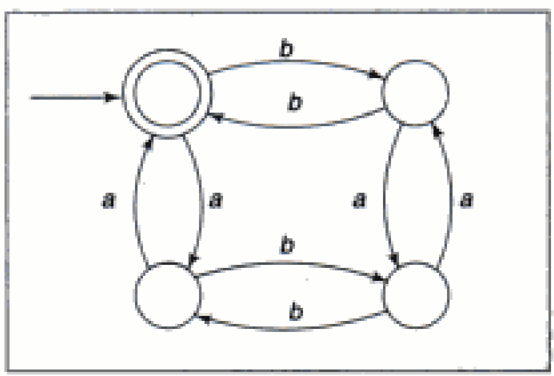

Fig. 6.5

27. Any given Transition graph has an equivalent

(a) regular expression
(b) DFSM
(c) NDFSM
(d) none of the above

28. The following CFG

$S \rightarrow aS \mid bS \mid a \mid b$

is equivalent to the regular expression

(a) (a*+b)* (b) (a+b)* (c) (a+b)(a+b)* (d) (a+b)*(a+b)

***29.** Any string of terminals that can be generated by the following CFG

S → XY
X → aX | bX | a
Y → Ya | Yb | a

(a) has at least one b
(b) should end in an 'a'
(c) has no consecutive a's or b's
(d) has at least two a's

***30.** The following CFG

S → aB | bA
A → b | aS | bAA
B → b | bS | aBB

generates strings of terminals that have

(a) equal number of a's and b's
(b) odd number of a's and odd number b's
(c) even number of a's and even number of b's
(d) odd number a's and even number of a's

31. Let L(G) denote the language generated by the grammar G. To prove set $A = L(G)$,

(a) it is enough to prove that an arbitrary member of A can be generated by grammar G
(b) it is enough to prove that an arbitrary string generated by G, belongs to set A
(c) both the above comments (a) and (b) are to be proved
(d) either of the above comments (a) or (b) is to be proved

***32.** The set $\{a^n b^n \mid n = 1,2,3 \ldots\}$ can be generated by the CFG

(a) S → ab | aSb
(b) S → aaSbb | ab
(c) S → ab | aSb | ε
(d) S → aaSbb | ab | aabb

33. Choose the correct statements.

(a) All languages can be generated by CFG.
(b) Any regular language has an equivalent CFG.
(c) Some non-regular languages can't be generated by any CFG.
(d) Some regular languages can't be generated by any CFG.

***34.** Which of the following CFG's can't be simulated by an FSM?

(a) S → Sa | a
(b) S → abX
X → cY
Y → d | aX
(c) S → aSb | ab
(d) None of the above

35. CFG is not closed under

(a) union (b) Kleene star (c) complementation (d) product

36. The set $A = \{a^n b^n a^n \mid n = 1, 2, 3, \ldots\}$ is an example of a grammar that is

(a) regular (b) context free

(c) not context free (d) none of the above

37. Let $L1 = \{a^n b^n a^m \mid m, n = 1, 2, 3, \ldots\}$

$L2 = \{a^n b^m a^m \mid m, n = 1, 2, 3, \ldots\}$

$L3 = \{a^n b^n a^n \mid n = 1, 2, 3, \ldots\}$

Choose the correct statements.

(a) $L3 = L1 \cap L2$

(b) L1 and L2 are CFL but L3 is not a CFL

(c) L1 and L2 are not CFL but L3 is a CFL

(d) L1 is a subset of L3

38. $L = \{a^n b^n a^n \mid n = 1, 2, 3, \ldots\}$ is an example of a language that is

(a) context free

(b) not context free

(c) not context free but whose complement is CF

(d) context free but whose complement is not CF

39. The intersection of a CFL and a regular language

(a) need not be regular (b) need not be context free

(c) is always regular (d) is always CF

40. A PDM behaves like an FSM when the number of auxiliary memory it has is

(a) 0 (b) 1 (c) 2 (d) none of the above

41. A PDM behaves like a TM when the number of auxiliary memory it has is

(a) 0 (b) 1 or more (c) 2 or more (d) none of the above

42. Choose the correct statements.

(a) The power of DFSM and NDFSM are the same.

(b) The power of DFSM and NDFSM are different.

(c) The power of DPDM and NDPDM are different.

(d) The power of DPDM and NDPDM are the same.

43. Which of the following is accepted by an NDPDM, but not by a DPDM?

(a) All strings in which a given symbol is present at least twice.

(b) Even palindromes (i.e. palindromes made up of even number of terminals).

(c) Strings ending with a particular terminal.

(d) None of the above

44. CSG can be recognized by a

(a) FSM (b) DPDM

(c) NDPDM (d) linearly bounded memory machine

45. Choose the correct statements.

(a) An FSM with 1 stack is more powerful than an FSM with no stack.

(b) An FSM with 2 stacks is more powerful than a FSM with 1 stack.

(c) An FSM with 3 stacks is more powerful than an FSM with 2 stacks.

(d) All of these.

46. Choose the correct statements.

(a) An FSM with 2 stacks is as powerful as a TM.

(b) DFSM and NDFSM have the same power.

(c) A DFSM with 1 stack and an NDFSM with 1 stack have the same power.

(d) A DFSM with 2 stacks and an NDFSM with 2 stacks have the same power.

47. Bounded minimalization is a technique for

(a) proving whether a primitive recursive function is Turning computable

(b) proving whether a primitive recursive function is a total function

(c) generating primitive recursive functions

(d) generating partial recursive functions

48. Which of the following is not primitive recursive but computable?

(a) Carnot function (b) Riemann function

(c) Bounded function (d) Ackermann function

49. Which of the following is not primitive recursive but partially recursive?

(a) Carnot function (b) Rieman function

(c) Bounded function (d) Ackermann function

50. Choose the correct statements.

(a) A total recursive function is also a partial recursive function.

(b) A partial recursive function is also a total recursive function.

(c) A partial recursive function is also a primitive recursive function.

(d) A primitive recursive function is also a partial recursive function.

51. A language L for which there exists a TM, T, that accepts every word in L and either rejects or loops for every word that is not in L, is said to be

(a) recursive (b) recursively enumerable

(c) NP-HARD (d) none of the above

52. Choose the correct statements.

(a) $L = \{a^n b^n a^n \mid n = 1, 2, 3, \ldots\}$ is recursively enumerable.

(b) Recursive languages are closed under union.

(c) Every recursive language is recursively enumerable.

(d) Recursive languages are closed under intersection.

53. Choose the correct statements.

(a) Set of recursively enumerable languages is closed under union.

(b) If a language and its complement are both regular, then the language must be recursive.

(c) Recursive languages are closed under complementation.

(d) None of the above.

54. Pick the correct answers.

Universal TM influenced the concept of

(a) stored-program computers

(b) interpretive implementation of programming languages

(c) computability

(d) none of the above

55. The number of internal states of a UTM should be at least

(a) 1 (b) 2 (c) 3 (d) 4

56. The number of symbols necessary to simulate a TM with m symbols and n states is

(a) $m + n$ (b) $8mn + 4m$ (c) mn (d) $4mn + m$

57. Any TM with m symbols and n states can be simulated by another TM with just 2 symbols and less than

(a) $8mn$ states (b) $4mn + 8$ states (c) $8mn + 4$ states (d) mn states

58. The statement — "A TM can't solve halting problem" is

(a) true (b) false

(c) still an open question (d) none of the above

59. If there exists a TM which when applied to any problem in the class, terminates if the correct answer is yes, and, may or may not terminate otherwise is said to be

(a) stable (b) unsolvable (c) partially solvable (d) unstable

60. The number of states of the FSM, required to simulate the behaviour of a computer, with a memory capable of storing 'm' words, each of length 'n' bits is

(a) $m \times 2^n$ (b) 2^{mn} (c) 2^{m+n} (d) none of the above

61. The vernacular language English, if considered a formal language, is a

(a) regular language (b) context free language

(c) context sensitive language (d) none of the above

***62.** Let P, Q, and R be three languages. If P and R are regular and if PQ = R, then

(a) Q has to be regular (b) Q cannot be regular

(c) Q need not be regular (d) Q has to be a CFL

63. Consider the grammar

```
S → PQ | SQ | PS
P → x
Q → y
```

To get a string of n terminals, the number of productions to be used is

(a) n^2 (b) $n + 1$ (c) $2n$ (d) $2n - 1$

64. Choose the correct statements.

A class of languages that is closed under

(a) union and complementation has to be closed under intersection

(b) intersection and complementation has to be closed under union

(c) union and intersection has to be closed under complementation

(d) all of the above

65. The following grammar is

$S \rightarrow a\alpha b \mid b\alpha c \mid aB$

$S \rightarrow aS \mid b$

$S \rightarrow \alpha bb \mid ab$

$b\alpha \rightarrow bdb \mid b$

(a) context free (b) regular (c) context sensitive (d) LR (k)

***66.** Which of the following definitions generates the same language as L, where

$L = \{x^n y^n,\ n \geq 1\}$?

I. $E \rightarrow xEy \mid xy$

II. $xy \mid x^+xyy^+$

III. x^+y^+

(a) **I** only (b) **I** and **II** (c) **II** and **III** (d) **II** only

***67.** A finite state machine with the following state table has a single input x and a single output z.

Present state	*Next state, z*	
	$x = 1$	$x = 0$
A	D, 0	B, 0
B	B, 1	C, 1
C	B, 0	D, 1
D	B, 1	C, 0

If the initial state is unknown, then the shortest input sequence to reach the final state C is

(a) 01 (b) 10 (c) 101 (d) 110

***68.** Let $A = \{0, 1\}$ and $L = A^*$. Let $R = \{0^n 1^n,\ n>0\}$. The languages L U R and R are respectively

(a) regular, regular (b) not regular, regular

(c) regular, not regular (c) not regular, not regular

***69.** Which of the following conversion is not possible algorithmically?

(a) Regular grammar to context free grammar

(b) Non-deterministic FSA to deterministic FSA

(c) Non-deterministic PDA to deterministic PDA

(d) Non-deterministic Turing machine to deterministic Turing machine

***70.** An FSM can be used to add two given integers. This remark is

(a) true (b) false (c) may be true (d) none of the above

***71.** A CFG is said to be in Chomsky Normal Form (CNF), if all the productions are of the form `A → BC or A → a`. Let `G` be a CFG in CNF. To derive a string of terminals of length x, the number of productions to be used is

(a) $2x - 1$ (b) $2x$ (c) $2x + 1$ (d) 2^x

Answers

1. c	2. d	3. a, b, c	4. b	5. a, b, c, d
6. c	7. d	8. b	9. a, b, c	10. a, b, d
11. c	12. a	13. a	14. a, b, c	15. b
16. c	17. c	18. a	19. a	20. a, b, c
21. d	22. d	23. d	24. c	25. a, d
26. c	27. a, b, c	28. b, c, d	29. d	30. a
31. c	32. a, d	33. b, c	34. c	35. c
36. c	37. a, b	38. b, c	39. c, d	40. a
41. c	42. a, c	43. b	44. d	45. a, b
46. a, b, d	47. c	48. d	49. d	50. a, d
51. b	52. a, b, c, d	53. a, b, c	54. a, b, c	55. b
56. d	57. a	58. a	59. c	60. b
61. b	62. c	63. d	64. a, b	65. c
66. a	67. b	68. c	69. c	70. b
71. a				

Explanations

6. DFSM is a special case of NDFSM. Corresponding to any given NDFSM, one can construct an equivalent DFSM. Corresponding to any given DFSM, one can construct an equivalent NDFSM. So they are equally powerful.

9. Strings of odd number of zeroes can be generated by the regular expression `(00)*0`. Pumping lemma can be used to prove the non-regularity of the other options.

10. Two regular expressions `R1` and `R2` are equivalent if any string that can be generated by `R1` can be generated by `R2` and vice-versa. In option (c), `(ab)*` will generate `abab`, which is not of the form $a^n b^n$ (because a's and b's should come together). All other options are correct (check it out!).

12. Pigeon-hole principle is that if 'n' balls are to be put in 'm' boxes, then at least one box will have more than one ball if $n>m$. Though this is obvious, still powerful.

13. That's why it can't recognize strings of equal number of a's and b's, well-formedness of nested parenthesis etc.

17. Here the `final` state and the `start` state are one and the same. No transition is there. But by definition, there is an (implicit) ε-transition from any state to itself. So, the only string that could be accepted is ε.

22. Refer Qn. 17. In the second diagram, the final state is unreachable from the `start` state. So not even ε could be accepted.

24. Let 011011 be the input to the FSM and let it be fed from the right (i.e., least significant digit first). If we add 1 to 011011 we should get 011100. But did we obtain it? Whenever we add 1 to an 1, we make it 0 and carry 1 to the next stage (state) and repeat the process. If we add 1 to a 0, then first make it 1 and all the more significant digits will remain the same, i.e., a 0 will be 0 and an 1 will be 1. That's what the given machine does. Hence the answer is (c).

26. Here the initial and the final states are one and the same. If you carefully examine the transition diagram, to move right you have to consume a '*b*', to move left a '*b*', to go up an '*a*' and to go down an '*a*'. Whenever we move right, we have to move left at some stage or the other, to get back to the initial-cum-final state. This implies, a '*b*' essentially has an associated another '*b*'. Same is the case with '*a*' (since any up (down) has a corresponding down (up)). So, even number of *a*'s and *b*'s have to be present.

29. S is the `start` state. X→a, Y→a are the only productions that could terminate a string derivable from X and Y respectively. So at least two *a*'s have to come anyway. Hence the answer is (d).

30. We have S → aB → aaBB → aabB → aabb.

So (b) is wrong. We have

S → aB → ab

So (c) is wrong.

A careful observation of the productions will reveal a similarity. Change A to B, B to A, a to b and b to a. The new set of productions will be the same as the original set. So (d) is false and (a) is the correct answer.

32. Option (b) is wrong because it can't generate aabb (in fact any even power). Option (c) is wrong since it generates ε also. Both (a) and (d) are correct.

34. Option (c) generates the set $\{a^n b^n,\ n = 1, 2, 3, \ldots\}$ which is not regular. Options (a) and (b) being left linear and right linear respectively, should have equivalent regular expressions.

60. Totally there are *mn* bits. Each bit will be in one of the two possible states – 1 or 0. So the entire memory made up of *mn* bits will be in one of the possible 2^{mn} states.

62. For example, $P = a^*;\ Q = a^n b^n b^*;\ R = PQ = a^* b^*$

64. The first two options can be proved to be correct using De Morgan's laws. Option (c) can be disproved by the following counter-example. Let the universal set U be {a, b, c, d,}. Let A = {{a}, {d}, {a, d}, {b, d}, {a, b, d}, {}}. A is closed under union and intersection but is not closed under complementation. For example complement of {a, d} is {b, c}, which is not a member of A.

66. II generates strings like xxyyy, which are not supposed to be. III generates strings like xyy, which are not supposed to be. I can be verified to generate all the strings in L and only those.

67. Draw the transition diagram and verify that the string `10` from `A`, leads to `C`.

68. L is the set of all possible strings made up of 0's and 1's (including the null string). So, L U R is L, which can be generated by the regular expression `(a+b)*`, and hence a regular language. `R` is not a regular expression. This can be proved by using Pumping Lemma or simply by the fact that finite state automata, that recognizes regular expressions, has no memory to record the number of 0's or 1's it has scanned. Without this information 0^n1^n cannot be recognized.

69. In general, a language (or equivalently the machine that recognizes it) cannot be converted to another language that is less powerful.

70. FSM is basically a language acceptor. As such, it does not have any output capability. So it cannot add and output the result.

71. This can be proved using induction.