Chapter 4

Mathematical Foundations of Computer Science

***1.** A class of 30 students occupy a classroom containing 5 rows of seats, with 8 seats in each row. If the students seat themselves at random, the probability that the sixth seat in the fifth row will be empty is

(a) 1/5 (b) 1/3 (c) 1/4 (d) 2/5

***2.** The probability that a number selected at random between 100 and 999 (both inclusive) will not contain the digit 7 is

(a) 16/25 (b) $(9/10)^3$ (c) 27/75 (d) 18/25

***3.** 0.152525252... is same as

(a) 52/99 (b) 151/990 (c) 51/99 (d) none of the above

***4.** A class is composed of 2 brothers and 6 other boys. In how many ways can all the boys be seated at a round table so that the two brothers are not seated together?

(a) 3600 (b) 3000 (c) 2600 (d) 2050

***5.** The n^{th} order difference of a polynomial of degree n is

(a) zero (b) one (c) some constant (d) undefined

***6.** Each coefficient in the equation $ax^2 + bx + c = 0$ is determined by throwing an ordinary die. The probability that the equation will have real roots is

(a) 57/216 (b) 27/216 (c) 53/216 (d) 43/216

***7.** The sum of all numbers greater than 10,000 formed by using the digits 0, 2, 4, 6, 8, no digit being repeated in any number, is

(a) 5199960 (b) 2742790 (c) 2449002 (d) 8411420

***8.** For a game in which 2 partners oppose 2 other partners, six men are available. If every possible pair must play against every other pair, the number of games to be played is

(a) 36 (b) 45 (c) 42 (d) 90

***9.** Let the elements g, h belong to a group G. If $O(h)$ is 2, then $O(ghg^{-1})$ is

(a) 0 (b) 1 (c) 2 (d) 4

10. At any time, the total number of persons on earth who have shaken hands an odd number of times has to be

(a) an even number (b) an odd number (c) a prime number (d) a perfect square

11. Which of the following are irrational numbers?

(a) $\sqrt{2}$ (b) e (c) 10.2 (d) 1.25252525...

12. The function $f(x) = |x/(x+1)|$

(a) is less than 1, for all x
(b) equals $f(-x)$
(c) equals $1 - f(1/x)$
(d) none of the above

***13.** The domain of the function log (log sin (x)) is

(a) $0 < x < \pi$
(b) $2n\pi < x < (2n + 1)\pi$, $n \in N$
(c) empty set
(d) none of the above

***14.** The system of equations

$$x + 2y + 3z = 4$$
$$x + \lambda y + 2z = 3$$
$$x + 4y + \mu z = 3$$

has infinite number of solutions if

(a) $\lambda = 2; \mu = 3$
(b) $\lambda = 2; \mu = 4$
(c) $3\lambda = 2\mu$
(d) none of the above

***15.** Let R be a symmetric and transitive relation on a set A. Then

(a) R is reflexive and hence an equivalence relation
(b) R is reflexive and hence a partial order
(c) R is not reflexive and hence not an equivalence relation
(d) none of the above

***16.** The number of elements in the power set of the set

$\{\{\{\}\}, 1, \{2, 3\}\}$ is

(a) 2 (b) 4 (c) 8 (d) 3

***17.** If $4(\log_9 3) + 9(\log_2 4) = 10(\log_x 81)$, then x is

(a) 2
(b) e
(c) 7
(d) none of the above

***18.** The length of the longest pole that can be made inside a hall of length 18m, breadth 6m, and height 4.5m is

(a) 17.5m (b) 19.5m (c) 20m (d) 18.25m

***19.** Six x's have to be placed in the squares in the adjacent figure, such that each row contains at least one x. This can be done in

(a) 160 ways (b) 180 ways
(c) 170 ways (d) 26 ways

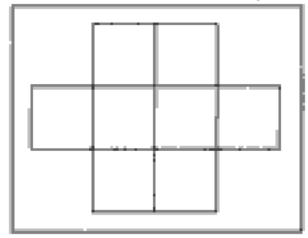


Fig. 4.1

***20.** Out of 100 students, 10 students used to drink milk(M), coffee(C) and tea(T); 20 M and C; 30 C and T; 25 M and T; 12 M only; 5 C only and 8 T only. The number of students who did not drink any of these is

(a) 18 (b) 24 (c) 20 (d) 16

21. Given the relation $R = \{(1, 2), (2, 3)\}$. The minimum number of ordered pairs that must be added to this set so that the enlarged relation is reflexive, symmetric and transitive is

(a) 4 (b) 5 (c) 6 (d) 7

22. A box contains 2 black, 4 white and 3 red balls. One ball is drawn at random from the box and kept aside. From the remaining balls in the box, another ball is drawn at random and kept beside the first. This process is repeated till all the balls are drawn from the box. The probability that the balls drawn are in the sequence 2 black, 4 white, and 3 red is

(a) 1/1260 (b) 17/1260 (c) 7/1260 (d) 13/1260

23. The range of the function $f(x) = x^2 / (1 + x^2)$ is

(a) $(-\infty, +\infty)$ (b) $(0, \infty)$ (c) $(-\infty, 0]$ (d) $[0, 1)$

24. In calculating the mean and variance of 10 readings, a student wrongly used 52 instead of the correct figure 25. If the mean he obtained was 45, then the correct mean is

(a) 47.3 (b) 43.7 (c) 42.3 (d) impossible to find

25. Refer Qn. 24. If the variance he obtained was 16, then the correct variance is

(a) 43.8 (b) 47.3
(c) 42.3 (d) impossible to find

***26.** If ${}^nC_{r-1} = 36$; ${}^nC_r = 84$ and ${}^nC_{r+1} = 126$, then the value of 'r' is

(a) 9 (b) 6 (c) 5 (d) 3

27. In the interval $[0, \pi]$, the equation $x = \cos(x)$ has

(a) no solution (b) exactly one solution
(c) exactly two solutions (d) an infinite number of solutions

***28.** Ten different letters are given. Five letter words are formed from these given letters. The number of words having at least one letter repeated is

(a) 99748 (b) 87882 (c) 92182 (d) 69760

***29.** The value of the expression ${}^{47}C_4 + \sum_{j=1}^{5} {}^{(52-j)}C_3$ is equal to

(a) 47! / 52! (b) 46! / 52! (c) ${}^{52}C_4$ (d) ${}^{52}C_{47}$

***30.** The rank of the following $(n + 1) \times (n + 1)$ matrix, where a is a real number is

$$\begin{pmatrix} 1 & a & a^2 & \ldots\ldots & a^n \\ 1 & a & a^2 & \ldots\ldots & a^n \\ . & . & . & & \\ . & . & . & & \\ 1 & a & a^2 & \ldots\ldots & a^n \end{pmatrix}$$

(a) 1 (b) 2
(c) n (d) dependent on the value of a.

31. Let 'S' be the standard deviation of 'n' numbers. If each of the 'n' numbers is multiplied by a constant C, then the new standard deviation will be

(a) $C \times S$ (b) $S\sqrt{C}$ (c) S (d) none of the above

***32.** Let A be a finite set of size 'n'. The number of elements in the power set of $A \times A$ is

(a) 2^{2^n} (b) 2^{n^2} (c) $(2^n)^2$ (d) $(2^2)^n$

***33.** Probability of an event A happening is 0.4. Probability that in 3 independent trials, event A happens at least once is

(a) 0.064 (b) 0.144 (c) 0.784 (d) 0.4

***34.** If x, y are two real numbers such that $x > 0$ and $xy = 1$, then $x + y$ can't be less than

(a) 1.5 (b) 1.9 (c) 1.75 (d) 2.0

***35.** Let $f(x + y) = f(x) + f(y)$, for all x, y. If $f(x)$ is continuous at $x = 0$, then

(a) f is continuous at all points
(b) the number of points of discontinuity of f can't be infinite
(c) the number of points of discontinuity of f must be infinite
(d) none of the above.

***36.** Let $f(x + y) = f(x)f(y)$, for all x, y. If $f(5) = 2$ and $f'(0) = 3$, Then $f'(5)$ is equal to

(a) 1 (b) 5 (c) 6 (d) -1

37. In numerical methods, accuracy refers to the

(a) number of significant figures representing a quantity
(b) spread in repeated readings of an instrument in measuring a particular physical quantity
(c) proximity of an approximate number or measurement to the true value it is supposed to represent
(d) all of the above

***38.** Suppose $A_1, A_2, \ldots A_{30}$ are 30 sets, each with 5 elements, and $B_1, B_2, \ldots B_n$ are 'n' sets, each with 3 elements.

Let $\bigcup_{i=1}^{30} A_i = \bigcup_{j=1}^{n} B_j = S$.

Each element of S, belongs to exactly 10 of the A_i's and to exactly 9 of the B_j's. Then 'n' is

(a) 25 (b) 45 (c) 40 (d) 20

39. Which of the following remarks about an ill-conditioned system of equations are true?

(a) Small change in coefficient will result in large change in solution.

(b) A wide range of solutions can approximately satisfy the equations.

(c) If slope of two lines are almost same, they make up an ill-conditioned system of equations.

(d) None of the above.

***40.** If the cube roots of unity are 1, ω, ω^2, then the roots of the equation $(x-1)^3 + 8 = 0$, are

(a) -1; $1 + 2\omega$, $1 + 2\omega^2$ (b) 1, $1 - 2\omega$, $1 - 2\omega^2$

(c) -1, $1 - 2\omega$, $1 - 2\omega^2$ (d) -1, $-1 + 2\omega$, $-1 + 2\omega^2$

***41.** $f(x)$ and $g(x)$ are two functions differentiable in $[0, 1]$ such that $f(0) = 2$; $g(0) = 0$; $f(1) = 6$; and $g(1) = 2$. Then there must exist a constant C in

(a) $(0, 1)$, such that $f'(C) = 2g'(C)$

(b) $[0, 1]$, such that $f'(C) = 2g'(C)$

(c) $(0, 1)$, such that $2f'(C) = g'(C)$

(d) $[0, 1]$, such that $2f'(C) = g'(C)$

***42.** Let f be a one-to-one function with domain $\{x, y, z\}$ and range $\{1, 2, 3\}$. It is given that exactly one of the following statements is true and the remaining 2 are false:

$f(x) = 1$

$f(y) \neq 1$

$f(z) \neq 2$

Then $f^{-1}(1)$ equals

(a) 2 (b) x (c) y (d) z

***43.** Let f be a twice differentiable function such that

$f''(x) = -f(x)$ and $f'(x) = g(x)$. Let $h(x) = (f(x))^2 + (g(x))^2$. If $h(5) = 11$, then $h(10)$ is

(a) 8 (b) 9 (c) 10 (d) 11

44. If A and B are two events such that $P(A) > 0$ and $P(B) \neq 1$, then $P(\bar{A}/\bar{B})$ equals

(a) $(1 - P(A \cup B)) / P(\bar{B})$ (b) $(1 - P(A \cup B)) / P(B)$

(c) $(1 - P(A \cap B)) / P(\bar{B})$ (d) $(1 - P(A \cap B)) / P(B)$

***45.** i^i, where i is $\sqrt{-1}$, is

(a) a pure imaginary number (b) a complex number

(c) an integer (d) a real number

46. If p, q, r are three real numbers, then

(a) $\max(p, q) < \max(p, q, r)$ (b) $\max(p, q) = (p + q + |p - q|) / 2$

(c) $\max(p, q) < \min(p, q, r)$ (d) none of the above

***47.** The number of 1's in the binary representation of $(3 \times 4096 + 15 \times 256 + 5 \times 16 + 3)$ is

(a) 8 (b) 9 (c) 10 (d) 12

48. A determinant is chosen at random from the set of all determinants of order 2 with each element either 0 or 1 only. The probability that the value of the chosen determinant is positive is

(a) 1/2 (b) 2/7 (c) 3/16 (d) 7/16

***49.** The number of permutations of 'n' different things taken not more than 'r' at a time, with repetitions being allowed, is

(a) $(n^r - 1) / (n - 1)$ (b) $(n^r - 1) / (n - 1)!$
(c) $n(n^r - 1) / (n - 1)$ (d) $(n^r - 1) / n!$

50. A relation R is defined in $N \times N$, such that $(a, b)\ R\ (c, d)$ iff $a + d = b + c$. The relation R is

(a) reflexive but not transitive (b) reflexive and transitive, but not symmetric
(c) an equivalence relation (d) a partial order

***51.** If $\log_5 10 = \log_7 x(\log_n m)$, then the values of x, m, n are

(a) 10, 7, 5 (b) −1, 2, 3 (c) 7, 5, 3 (d) 7, 5, 8

***52.** If $\sqrt{5} + \sqrt{7} + i$, is one of the roots of the equation $f(x) = 0$ with rational coefficients, then the degree of the given equation can't be less than

(a) 5 (b) 6 (c) 7 (d) 8

***53.** Consider the equation $x^7 - 2x^5 + 7x^4 + x^3 - 9 = 0$. The number of imaginary roots will be at least

(a) 2 (b) 3 (c) 4 (d) 5

***54.** If $f(a)$ and $f(b)$ are of the same sign, then the equation $f(x) = 0$

(a) has either no root or even number of roots between a and b
(b) must have at least one root between a and b
(c) has either no root or odd number of roots between a and b
(d) has odd number of roots between a and b

***55.** The equation $x^5 + x^3 - 8x - 5 = 0$ has

(a) exactly 3 real roots and 2 complex roots
(b) no complex root
(c) no real root
(d) exactly 2 real roots and 3 complex roots

***56.** Any polynomial of even degree in which the last term is negative and the coefficient of the highest power is positive, has at least

(a) 2 positive roots (b) 2 negative roots
(c) 1 positive root and 1 negative root (d) 2 positive and 1 negative root

***57.** When the polynomial $f(x)$ is divided by $(x - \alpha)(x - \beta)$, $\alpha \neq \beta$ then the remainder is given by

(a) $((x - \beta) f(\alpha) - (x - \alpha) f(\beta)) / (\alpha - \beta)$ (b) $((x - \alpha) f(\beta) - (x - \beta) f(\alpha)) / (\alpha - \beta)$
(c) $(f(\alpha) - f(\beta)) / (\alpha - \beta)$ (d) $((x - \alpha) f(\beta) + (x - \beta) f(\alpha)) / (\alpha - \beta)$

***58.** log 0 is

(a) $-\infty$ (b) $+\infty$
(c) depends on the base (d) undefined

*59. If $a_1, a_2, \ldots, a_n$ are the roots of the equation $x^n + nax - b = 0$, then $(a_1 - a_2)(a_1 - a_3) \ldots (a_1 - a_n)$ equals

(a) $n(a + a_1^{n-1})$ (b) $(a + a_1^{n-1})/n$ (c) $n(a - a_1^{n-1})$ (d) $(a - a_1^{n-1})/n$

60. The set of all natural numbers is not closed with respect to

(a) subtraction (b) division (c) addition (d) multiplication

*61. If $|a - b| < n$ and $|b - c| < m$, then $|a - c|$ is

(a) $< n + m$ (b) < maximum of m, n

(c) < minimum of m, n (d) $< mn$

62. The domain of the function $1/\sqrt{(1-x)(x-2)}$ is

(a) $(1, \infty)$ (b) $(1, 2)$ (c) $(2, \infty)$ (d) $(0, 2)$

*63. A and B play a coin tossing game. They toss a coin alternately. The first one to get a head wins. If A starts, the probability of A winning is

(a) 1/3 (b) 1/2 (c) 2/3 (d) 1/4

*64. The number of trailing zeroes in 200! (i.e., factorial of 200) is

(a) 49 (b) 40 (c) 48 (d) 52

*65. The determinant of a matrix has 720 terms (in the unsimplified form). The order of the matrix is

(a) 5 (b) 6 (c) 7 (d) 8

66. The error in using Simpson's rule is of the order

(a) h^2 (b) h^3 (c) h^4 (d) h^5

*67. The domain of the function $1/\sqrt{|x| - x}$ is

(a) $(-\infty, 0)$ (b) $(0, \infty)$ (c) $(0, x)$ (d) $(0, 1)$

*68. A bag contains 10 white balls and 15 black balls. Two balls are drawn in succession. The probability that one of them is black and the other white is

(a) 2/3 (b) 4/5 (c) 1/2 (d) 1/3

*69. The iteration formula to find the square root of a positive real number b, using the Newton-Raphson method is

(a) $x_{k+1} = 3\,(x_k + b)\,/\,2\,x_k$ (b) $x_{k+1} = (x_k^2 + b)/2\,x_k$

(c) $x_{k+1} = x_k - 2\,x_k\,/(x_k^2 + b)$ (d) none of the above

70. If $|x - 1| + |x - 2| + |x - 3| \geq 6$, then

(a) $x \leq 0$ or $x \geq 4$ (b) $1 \leq x \leq 3$ (c) $x \leq 3$ (d) $x \geq 1$

71. The number of real roots of the equation $|x|^2 - 3|x| + 2 = 0$ is

(a) 1 (b) 2 (c) 3 (d) 4

*72. $-20\sqrt{-\sqrt{20 - \sqrt{\cdots}}}$ equals

(a) -4 (b) -8 (c) -20 (d) -35

***73.** Two events A and B have probabilities 0.25 and 0.5 respectively. The probability that both A and B occur simultaneously is 0.14. Then the probability that neither A nor B occurs is

(a) 0.25 (b) 0.75 (c) 0.39 (d) 0.11

***74.** A function $f(x)$ differentiable in the interval $0 \leq x \leq 5$, is such that $f(0) = 4$ and $f(5) = -1$. If $g(x) = f(x) / (x + 1)$, then there exists some constant C, $0 < C < 5$ such that $g'(C)$ equals

(a) $-2/5$ (b) $2/5$ (c) $-3/5$ (d) $-5/6$

75. Let S be an infinite set and $S1, S2, \ldots, Sn$ be sets such that $S1 \cup S2 \cup \ldots \cup Sn = S$. Then,

(a) at least one of the sets Si is a finite set

(b) not more than one of the sets Si can be finite

(c) at least one of the sets Si should be infinite

(d) not more than one of the sets Si can be infinite

***76.** Let A and B be sets with cardinalities 'm' and 'n' respectively. The number of possible one to one mappings (injections) from A to B, when $m < n$, is

(a) m^n (b) ${}^mC_n$ (c) ${}^nP_m$ (d) ${}^mP_2$

***77.** Choose the correct answers.

The set $\{1, 2, 3\}$ is equal to

(a) $\{2, 1, 3\}$ (b) $\{3, 2, 1\}$ (c) $\{1, 2, 3, 4\}$ (d) $\{1, 2, 3, 1\}$

78. Let $A = \{1, \{2\}, 3\}$.

Choose the correct option.

(a) $1 \in A$ (b) $\{2\} \subset A$ (c) $\phi \in A$ (d) $\phi \subset A$

79. Choose the correct answers.

If A, B, C are three sets, then

(a) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ (b) $(A - B) - C = (A - C) - (B - C)$

(c) $(A \times B) \times C = A \times (B \times C)$ (d) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

***80.** In the set of integers, a relation R is defined as aRb, if and only if $b = |a|$. This relation is

(a) reflexive (b) irreflexive (c) symmetric (d) anti-symmetric

***81.** Let $S = \{1, 2, 3, 4\}$. A relation R defined in S as, $R = \{(1, 2), (4, 3), (2, 2), (2, 1), (3, 1)\}$ is

(a) transitive (b) symmetric (c) anti-symmetric (d) none of the above

***82.** Let $A = \{1, 2, 3\}$. Which of the following relations are functions (mappings)?

(a) $\{(1, 2), (2, 3), (1, 3)\}$ (b) $\{(1, 2), (2, 2), (3, 2)\}$

(c) $\{(1, 2), (2, 1), (3, 3)\}$ (d) $\{(1, 2), (2, 3)\}$

83. Consider the mapping $f : X \to Y$. f is a bijection if and only if

(a) $f(x) = f(y) \Rightarrow x = y$, for all x, y (b) range of f is Y

(c) both *(a)* and *(b)* are true (d) the co-domain equals the range

84. For a function to be invertible, it has to be

(a) one-one (b) onto

(c) both one-one and onto (d) none of the above

85. The advantages of partial pivoting in the solution of a system of equations are

(a) division by zero can be avoided

(b) round-off errors can be minimized
(c) ill-conditioned system can be handled efficiently
(d) none of the above

***86.** Choose the correct statements.
(a) Any 7 integers chosen from 1 to 12 should have at least 2 of them summing up to 13.
(b) Any 11 integers chosen from 1 to 20 should have at least 2 numbers, such that one is a multiple of the other.
(c) 10 integers, 1 to 10 arranged at random in a circle should have at least 3 successive numbers summing up to greater than 16.
(d) None of the above.

87. Choose the correct statements.
(a) If two graphs $G1$ and $G2$ are isomorphic, then they should have the same number of vertices and edges.
(b) If two graphs have the same number of nodes and edges, they have to be isomorphic.
(c) Loops can't be present in an isomorphic graph.
(d) None of the above.

88. In any undirected graph, the sum of degrees of all the nodes
(a) must be even
(b) is twice the number of edges
(c) must be odd
(d) need not be even

89. $(PVQ) \wedge (P \to R) \wedge (Q \to S)$ is equivalent to
(a) $S \wedge R$ (b) $S \to R$ (c) $S \vee R$ (d) none of the above

90. Which of the following are tautologies?
(a) $((PVQ) \wedge Q) \leftrightarrow Q$ (b) $(P \vee (P \to Q)) \to P$
(c) $((PVQ) \wedge P) \to Q$ (d) $((PVQ) \wedge \neg P) \to Q$

91. Identify the valid conclusion from the premises $P \vee Q$, $Q \to R$, $P \to M$, $\neg M$
(a) $P \wedge (Q \vee R)$ (b) $P \wedge (Q \wedge R)$ (c) $R \wedge (P \vee Q)$ (d) $Q \wedge (P \vee R)$

92. T is a graph with 'n' vertices. If T is connected and has exactly $n-1$ edges, then
(a) T is a tree
(b) T contains no cycles
(c) every pair of vertices in T is connected by exactly one path
(d) the addition of a new edge will create a cycle.

***93.** If one has to obtain the roots of $x^2 - 2x + \log 2 = 0$ to four decimal places, log 2 should be given to the accuracy of approximately
(a) 6×10^{-5} (b) 7×10^{-6} (c) 8×10^{-5} (d) 9×10^{-7}

***94.** Choose the incorrect statement(s).
(a) The determinant of a matrix equals the sum of its eigen values.
(b) A matrix satisfies its characteristic equation.

(c) The sum of the principal diagonal elements of a matrix equals the sum of its eigen values.

(d) If a row of a matrix is same as one of its columns, its determinant value is 0.

***95.** M is a square matrix of order 'n' and its determinant value is 5. If all the elements of M are multiplied by 2, its determinant value becomes 40. The value of 'n' is

(a) 2 (b) 3 (c) 4 (d) 5

96. In a computer an n-digit integer $a_n\, a_{n-1} \ldots a_1$ is represented as $a_n\, a_{n-1} \ldots a_{r+1}\, 00 \ldots 0$. The error e is

(a) $0 \le e \le 10^{r-1}$ (b) $1 \le e \le 10^r - 1$ (c) $0 \le e \le 10^{r-1} - 1$ (d) $0 \le e \le 10^{r+1} - 1$

97. $1 - x^2/2! + x^4/4! - \ldots + (-1)^n\, x^{2n}/2n! + \ldots$ is the expansion of

(a) e^x (b) $\log x$ (c) $\cos x$ (d) $\sin x$

***98.** In the previous question, for 5-digit accuracy, if $|x| < \pi/2$, the number of terms in the series that should be considered is

(a) 5 (b) 7 (c) 9 (d) 10

99. Which of the following methods gives the least error when e^x is integrated from 0 to 0.4?

(a) Trapezoidal rule with the interval width as 0.2

(b) Trapezoidal rule with the interval width as 0.1

(c) Simpson's 1/3 rule with the interval width as 0.1

(d) Simpson's 1/3 rule with the interval width as 0.2

100. Which of the following laws doesn't hold good in finite precision floating point arithmetic?

(a) $a \times b = b \times a$ (b) $(a + b) + c = a + (b + c)$

(c) $a \times (b + c) = a \times b + a \times c$ (d) $a + a = 2 \times a$

101. Surplus variables are usually introduced in an LPP model

(a) if the demand is less than the available resource

(b) if the available resource is less than the demand

(c) if the demand is same as the available resource

(d) while solving the dual of the given primal

***102.** In an LPP model in its standard form, three of the constraints are

$$x_1 + x_2 \le 2$$
$$2x_1 + 2x_2 \le 3$$
$$3x_1 + 3x_2 \le 8$$

Removal of which of the constraints will not affect the optimality?

(a) II and III (b) I and II (c) I and III (d) I only

103. An LPP having 2 optimal solutions must have

(a) more than 3 constraints

(b) more than 2 optimal solutions

(c) even number of constraints

(d) none of the above

104. The number of iterations taken by simplex method for solving an LPP in its standard form, with 'm' equations and 'n' unknowns ($m < n$) can't exceed

(a) m_{C_n} (b) m_{P_n} (c) n_{C_m} (d) n_{P_m}

105. In the solution of an LPP using simplex method, the current cost of the objective function must

(a) increase in the next iteration
(b) can't decrease in the next iteration
(c) remain the same in the next iteration
(d) correspond to one of the corners of the convex region bound by the constraining inequations

106. If the cost of the objective function (of an LPP in its standard form) which corresponds to one of the corners of the convex region bound by the constraints, is greater than the cost corresponding to all its adjacent corners, then

(a) it is the optimal solution
(b) simplex method enters a cycle
(c) simplex method moves onto one of the adjacent corners
(d) simplex method terminates

107. Revised simplex method

(a) is conceptually same as the simplex method
(b) is a version of simplex method ideal for implementation in computer
(c) is a version of simplex method ideal for sensitivity analysis
(d) uses recursion instead of iteration to solve a given LPP

108. The dual simplex method starts with a

(a) feasible but super-optimal solution
(b) feasible but sub-optimal solution
(c) infeasible but super-optimal solution
(d) infeasible but sub-optimal solution

109. Which of the following simplex based techniques are ideal for sensitivity analysis?

(a) Revised simplex method (b) Parametric programming
(c) Dual simplex method (d) Big-M method

110. Choose the correct statements.

(a) It is computationally advantageous to solve a given LPP in its dual form, if the number of constraints in the primal form is more than the number of variables.
(b) The cost of the (primal) objective function corresponding to a feasible solution can't be greater than, the cost of the (dual) objective function corresponding to any of its feasible solution.
(c) It is computationally advantageous to solve a given LPP in its dual form, if the number of variables in the primal form is more than the number of constraints.

(d) The cost of the (primal) objective function corresponding to a feasible solution cannot be less than the cost of the (dual) objective function corresponding to any of its feasible solution.

111. Choose the correct statement(s).

(a) Addition of a new constraint to an LPP can never improve the optimal value

(b) Addition of a new variable can never decrease the optimal value

(c) Addition of a new constraint can never decrease the optimal value

(d) Addition of a new variable can never improve the optimal value

112. Changing the right hand side of the constraints and the coefficient of the cost function

(a) can't destroy the optimality of the solution

(b) can't destroy the feasibility of the solution

(c) can destroy the optimality and feasibility of the solution

(d) none of the above

113. Let A be the set of all non-singular matrices over real numbers and let * be the matrix multiplication operator. Then,

(a) A is closed under * but $<A, *>$ is not a semi-group

(b) $<A, *>$ is a semi-group but not a monoid

(c) $<A, *>$ is a monoid but not a group

(d) $<A, *>$ is a group but not an abelian group

114. Newton-Raphson method

(a) is not efficient in handling multiple roots

(b) has a slow rate of convergence

(c) should not be preferred if there is a point of inflexion in the vicinity of the root

(d) should not be preferred if the graph of the curve is almost parallel to the x-axis, in the vicinity of the root

115. In the bisection method for finding the roots of an equation, the approximate relative error is always

(a) greater than the relative error (b) equal to the relative error

(c) less than the relative error (d) none of the above

116. Trapezoidal rule gives the exact solution when the curve is

(a) concave towards the base line (b) convex towards the base line

(c) a straight line (d) none of the above

117. If a function $y' = f(x)$ has an inverse function, then $f(x)$ can't be

(a) symmetric about x-axis (b) an odd function

(c) symmetric about y-axis (d) none of the above

118. For what value of c, will the vector $i + cj$ be orthogonal to $2i - j$?

(a) 0 (b) 1 (c) 2 (d) 3

***119.** The solution of the differential equation $y'' + 3y' + 2y = 0$, is of the form

(a) $C_1e^{x} + C_2e^{2x}$ (b) $C_1e^{-x} + C_2e^{3x}$ (c) $C_1e^{-x} + C_2e^{-2x}$ (d) $C_1e^{-2x} + C_2e^{-x}$

***120.** If the proposition $\neg P \Rightarrow Q$ is true, then the truth value of the proposition $\neg P \vee (P \Rightarrow Q)$, is
(a) true (b) multi-valued (c) false (d) cannot be determined

***121.** The number of divisors of 600 (including 1 and 600) is
(a) 24 (b) 22 (c) 23 (d) 25

***122.** The determinant value of the matrix $\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 5 & 7 & 9 \end{pmatrix}$ is
(a) 12 (b) 16 (c) 42 (d) none of the above

***123.** Which of the following elementary operations may affect the rank of a matrix?
(a) Scalar multiplication
(b) Adding two rows
(c) Adding a row with the scalar multiple of another row
(d) None of the above

124. Which of the following will not form an abelian group?
(a) Addition over the set of natural numbers (b) Subtraction over the set of integers
(c) Multiplication over the set of integers (d) None of the above

***125.** A group has 11 elements. The number of proper sub-groups it can have is
(a) 0 (b) 11 (c) 5 (d) 4

***126.** Let A and B be two $n \times n$ real symmetric matrices. Then
(a) $AA^t = I$ (d) $A = A^{-1}$ (c) $AB = BA$ (d) $(AB)^t = BA$.

127. Backward Euler method for solving the differential equation $dy/dx = f(x, y)$, is specified by
(a) $y_{n+1} = y_n + hf(x_n, y_n)$ (b) $y_{n+1} = y_n + hf(x_{n+1}, y_{n+1})$
(c) $y_{n+1} = y_{n-1} + 2hf(x_n, y_n)$ (d) $y_{n+1} = (1 + h) f(x_{n+1}, y_{n+1})$

***128.** Let A and B be two arbitrary events. Then
(a) $P(A \cap B) = P(A) P(B)$ (b) $P(A \cup B) = P(A) + P(B)$
(c) $P(A/B) = P(A \cap B) + P(B)$ (d) $P(A \cup B) \le P(A) + P(B)$

129. The rank of the matrix

$$\begin{pmatrix} 0 & 0 & -3 \\ 9 & 3 & 5 \\ 3 & 1 & 1 \end{pmatrix} \text{ is}$$

(a) 0 (b) 1 (c) 2 (d) 3

***130.** (G, *) is an abelian group. Then
(a) $x = x^{-1}$, for any x belonging to G
(b) $x = x^2$, for any x belonging to G
(c) $(x*y)^2 = x^2*y^2$, for any x, y belonging to G
(d) G is of finite order

131. In a compact single dimensional array representation for lower triangular matrices (i.e., all the elements above the diagonal are zero), of size $n \times n$, non-zero elements (i.e., elements of the lower triangle) of each row are stored one after the other, starting from the first row. The index of the (i, j)th element of the lower triangular matrix in this new representation is

(a) $i + j$ (b) $i + j - 1$ (c) $j + i(i-1) / 2$ (d) $i+j(j-1) / 2$

***132.** The number of sub-strings (of all lengths) that can be formed from a character string of length n is

(a) n (b) n^2 (c) $n(n-1) / 2$ (d) $n(n+1) / 2$

133. In the set of natural numbers, the binary operators that are not associative and not commutative are

(a) addition (b) subtraction (c) multiplication (d) division

***134.** A relation R is defined as xRy, if $x \neq y$. This relation R is

(a) symmetric but not reflexive

(b) symmetric and transitive, but not reflexive

(c) not reflexive, not symmetric, and not transitive

(d) an equivalence relation

135. The number of subsets of $\{1, 2, \ldots, n\}$ of odd cardinality is

(a) dependent on the value of n (b) 2^{n-1}, if n is odd

(c) 2^{n-1}, if n is even (d) 2^{n-1}, for any value of n

***136.** The probability of an event B occurring is P. The probability that events A and B occur together is Q. The probability that A occurs, without B occurring, is R. Then the probability of A occurring is

(a) $P + Q + R$ (b) $P + Q - R$ (c) $Q + R$ (d) $P - Q - R$

137. Let A, B, C be independent events with probabilities 0.8, 0.5, 0.3. The probability of occurrence of at least one of these three is

(a) 0.3 (b) 0.93 (c) 0.12 (d) 0.07

138. The subset of a countable set

(a) has to be countable (b) may or may not be countable

(c) has to be finite (d) none of the above

139. Every element of some ring $(R, +, *)$ is such that $a * a = a$. This ring

(a) is commutative (b) is non-commutative

(c) may or may not be commutative (d) none of the above

140. For the M/G/1 queuing system, the arrival pattern and service time follows

(a) Poisson and Binomial (b) Binomial and Poisson

(c) General and Poisson (d) Poisson and General

141. Consider the set $\{1, 2, 3, 4, 6, 8, 12, 24\}$, together with the two binary operations LCM (Least Common Multiple) and GCD (Greatest Common Divisor). Which of the following does this algebraic structure represent?

(a) Group (b) Ring (c) Field (d) Lattice

***142.** The set {1, 2, 3, 4, 6, 8, 12, 24}, together with LCM as the binary operation is not a group, because

(a) it is not closed
(b) it is not associative
(c) identity does not exist
(d) inverse does not exist

***143.** The set {1, 2, 3, 4, 6, 8, 12, 24}, together with GCD as the binary operation is not a group, because

(a) it is not associative
(b) identity does not exist
(c) inverse is not unique
(d) inverse does not exist

144. The following set

(a) $Q(x) \rightarrow P(x) \vee \neg R(a)$
(b) $R(a) \vee \neg Q(a)$
(c) $Q(a)$
(d) $\neg P(y)$

where x and y are universally quantified variables, a is a constant and P, Q, R are monadic predicates, is

(a) consistent
(b) inconsistent
(c) may be consistent
(d) none of the above

***145.** Let X and Y be sets with cardinalities m and n respectively. If the number of possible functions that can be defined with domain X and co-domain Y is exactly 10, then

(a) $m = n = 10$
(b) $m = 1; n = 10$
(c) $m = 10; n = 1$
(d) $m = 5; n = 5$

146. Let $F: R^2 \rightarrow R^2$ be the mapping defined by $F(x, y) = (x/3, y/4)$. What will be the image of $x^2/9 + y^2/16 = 1$ under F?

(a) The circle $x^2 + y^2 = 1$
(b) The line $x/3 + y/4 = 1$
(c) The ellipse $x^2 + y^2 = 1$
(d) None of the above

***147.** A function g is defined as $g(x) = f(x)[f(x) + f(-x)]$. Which of the following remarks about the function g is right?

(a) g is even for all f
(b) g is odd for all f
(c) g is even if f is even
(d) g is even if f is odd

148. If $x \in [0, 1]$, and $f(x)$ and $g(x)$ are defined as $f(x) = \sin(\cos(x\pi/4))$ and $g(x) = \cos(\sin(x\pi/4))$, then

(a) f is monotonic increasing and g is monotonic decreasing
(b) f is monotonic increasing and g is monotonic increasing
(c) f is monotonic decreasing and g is monotonic decreasing
(d) f is monotonic decreasing and g is monotonic increasing

149. What is the total number of equivalent relations that can be defined on the set {1, 2, 3}?

(a) 8
(b) 64
(c) 5
(d) 3

***150.** Cube roots of unity form a cyclic group under multiplication. For this group,

(a) ω is the only generator
(b) ω, ω^2 are the only generators
(c) ω^2 is the only generator
(d) none of the above

***151.** The value of $\lim_{x \rightarrow 0} x \log x$ is

(a) $-\infty$
(b) ∞
(c) 1
(d) 0

***152.** The function $f(x)$ is continuous in [0, 1], such that $f(0) = -1$, $f(1/2) = 1$ and $f(1) = -1$. We can conclude that

(a) f attains the value zero at least twice in [0, 1]
(b) f attains the value zero exactly once in [0, 1]
(c) f is non-zero in [0, 1]
(d) f attains the value zero exactly twice in [0, 1]

***153.** The sum of the infinite series Σkx^k, where $-1 < x < 1$, is

(a) $x/(1-x)$ (b) $x/(1-x)^2$ (c) $x^2/(1-x)^2$ (d) $1/(1-x)$

***154.** Which of the following is not a linear transformation?

(a) $f : R^3 \to R^2$ defined by $f(x, y, z) = (x, z)$
(b) $f : R^3 \to R^3$ defined by $f(x, y, z) = (x, y - 1, z)$
(c) $f : R^2 \to R^2$ defined by $f(x, y) = (2x, y - x)$
(d) $f : R^2 \to R^2$ defined by $f(x, y) = (y, x)$

***155.** If the determinant of an $n \times n$ matrix A is zero, then

(a) rank of A is n
(b) rank of $A \leq n - 2$
(c) A has at least one zero eigen value
(d) the system of equations $Ax = 0$ has no solution other than the trivial solution

156. A is a 2×2 matrix with eigen values 2 and –3. The eigen values of the matrix A^2

(a) are 4 and –9 (b) are 2 and –3
(c) are 4 and 9 (d) cannot be determined from the given data

157. Among any $n + 1$ distinct positive integers less than or equal to $2n$, we can always find

(a) n numbers that are relatively prime to $2n$
(b) two numbers that are relatively prime to each other
(c) two prime numbers
(d) none of the above

158. Let X_1 and X_2 be any two unit vectors in R^3. The angle between the two planes $X_1 \cdot X = c$ and $X_2 \cdot X = 2c$, where c is a constant is given by

(a) 0 (b) $(X_1 \cdot X_2)/2$ (c) $X_1 \cdot X_2$ (d) none of the above

159. If $(x_1, x_2, x_3) \times (1, 3, 1) = (2, 1, 6)$, where x denotes the vector product, then (x_1, x_2, x_3) is given by

(a) (0, 1, 1) (b) $(m, 0, 1 - m)$ for all real m
(c) (–1, 2, –7) (d) there does not exist any such (x_1, x_2, x_3) in R^3

160. Which of the following is a cube root of the complex number $-27i$?

(a) $-3i$ (b) $-3/2\,(\sqrt{3} + i)$ (c) $-3/2\,(\sqrt{3} - i)$ (d) $3\,(\sqrt{3} - 1)$

161. Suppose a system has been evolved by extraterrestrial creatures having only 3 fingers. They use the figures 0, 1, 2 with 2 > 1 > 0. What will be the binary equivalent of 222 in this system?

(a) 101010 (b) 11000 (c) 10110 (d) 11010

162. If you want to retain the first 4 bits of given string of 8 bits and complement the last 4 bits then the correct mask and the operation should be

(a) `XOR` and `00001111`
(b) `XOR` and `11110000`
(c) `AND` and `00001111`
(d) `OR` and `11110000`

163. Which of the following logical operation almost resembles an arithmetic multiplication operation?

(a) `OR`
(b) `AND`
(c) `NOR`
(d) `XOR`

164. To change lower case to upper case letters in ASCII, the correct mask and operation should be (ASCII value of character A is 65 and character a is 97)

(a) `0100000` and `NOR`
(b) `0100000` and `OR`
(c) `0100000` and `NAND`
(d) `1011111` and `AND`

***165.** Consider the nested `for` loop

```
for I1 = 1 to N
 for I2 = 1 to I1
  for I3 = 1 to I2
   .
    .
     .
              for Ik = 1 to I(k-1)
                  PRINT I1, I2, I3, ..., Ik
```

How many times is the PRINT statement executed?

(a) k^N
(b) ${}^{(k+N-1)}C_k$
(c) ${}^{(k-N+1)}C_k$
(d) ${}^{(k-N-1)}C_k$

The next three questions are based on the following assumptions.

Let $f(x)$ represent the largest integer less than or equal to x. Let $g(x)$ represent the smallest integer greater than or equal to x.

166. Which of the following remark(s) will be true for any x?

(a) $g(x) = f(x) + 1$
(b) $f(x) = g(x)$
(c) $f(-x) = -g(x)$
(d) all of the above

167. Which of the following, lists $f(x)$, $g(x)$, x, $x-1$ and $x+1$ in a non-decreasing sequence?

(a) $x-1$, x, $g(x)$, $f(x)$, $x+1$
(b) $x-1$, x, $f(x)$, $g(x)$, $x+1$
(c) $x-1$, $f(x)$, x, $g(x)$, $x+1$
(d) $x-1$, x, $g(x)$, $x+1$, $f(x)$

168. x mod y is

(a) $x - yf(x)$
(b) $x - xf(y)$
(c) $x - yf(x/y)$
(d) $x - xf(x/y)$

169. For $n > 2$, the equation $x^n + y^n = z^n$, has no solution in positive integers. This is

(a) Fermat's last theorem
(b) Ramanujan Ecumenical theorem
(c) Newton's last theorem
(d) Fermat's last theorem

***170.** Which of the following values of x, y, and z, satisfies the equation $x^2 + y^2 = z^2$?

(a) $x = 121$, $y = 407$, $z = 887$
(b) $x = 777$, $y = 333$, $z = 101$
(c) $x = 7$, $y = 47$, $z = 57$
(d) None of the above

***171.** Which of the following values of x, y, and z, satisfies the equation $x^2 + y^2 = z^2$?

(a) x=122, y=406, z=887 (b) x=778, y=334, z=101

(c) x=8, y=47, z=58 (d) None of the above

***172.** According to the principle of logic, an implication and its contrapositive must be

(a) both true or both false (b) both true

(c) both false (d) none of the above

173. If an implication and its converse are both true, then they can be combined using

(a) if and only if (b) as long as (c) if...then...else (d) such that

***174.** Associate a code with each letter of the alphabet such that the code of an alphabet is its position in the alphabet set. For example, code of *c* is 3, *y* is 25 etc., What can you say about the word that is made up of alphabets whose product of the codes is 637245?

(a) It must have at least two B's (b) It must have at least two Y's

(c) It must have at least two Z's (d) It must have at least two Q's

***175.** Associate a code with each letter of the alphabet such that the code of a letter is its position in the alphabet set. For example, code of *c* is 3, *y* is 25 etc., Find the word that is made up of the letters whose product of the codes is 124950.

(a) Impossible to find (b) No such word exists

(c) The word is DELHI (d) None of the above

***176.** Associate a code with each letter of the alphabet such that the code of a letter is its position in the alphabet set. For example, code of *c* is 3, *y* is 25 etc., Find the word that is made up of the letters whose product of the codes is 3135.

(a) The word is CHESS (b) No such word exists

(c) More than one such word exist (d) None of the above

***177.** Associate a code with each letter of the alphabet such that the code of a letter is its position in the alphabet set. For example, code of *c* is 3, *y* is 25, etc. Find the word that is made up of the letters whose product of the codes is 1265.

(a) The word is WASP (b) No such word exists

(c) More than one such word exist (d) None of the above

***178.** What is the largest 10-digit integer, containing all the numerals 1,2,3,4,5,6,7,8,9,0, that is divisible by 4?

(a) 9876543210 (b) 987654204

(c) 9876543120 (d) None of the above

***179.** What is the largest 10-digit integer, containing all the numerals 1,2,3,4,5,6,7,8,9,0, that is divisible by 8?

(a) 9876543210 (b) 987654204

(c) 9876543120 (d) None of the above

***180.** What is the smallest 10-digit positive integer, containing all the numerals 1,2,3,4,5,6,7,8,9,0, that is divisible by 8?

(a) 1023456789 (b) 01234567968

(c) 1023457986 (d) None of the above

***181.** What is the largest 10-digit integer, containing all the numerals 1,2,3,4,5,6,7,8,9,0, that is divisible by 11?

(a) 9876543210 (b) 987654204 (c) 9876524130 (d) 9876543120

182. Manoj had 4 pairs of identical black socks and 5 pairs of identical green socks in a box. With his eyes closed, he took them out one by one. How many socks should he take out before he has a matching pair? (Assume that what is taken out is not put back.)

(a) 3 (b) 5 (c) 6 (d) 10

183. Ramu had 2 pairs of identical yellow socks, 3 pairs of identical blue socks, 4 pairs of identical green socks, and 5 pairs of identical red socks in a box. With his eyes closed, he took them out one by one. How many socks should he take out before he is guaranteed to have a pair of red socks? (Assume that what is taken out is not put back.)

(a) 5 (b) 6 (c) 15 (d) 20

184. .Manoj had 4 pairs of black shoes and 5 pairs of green shoes in a box. With his eyes closed, he took them out one by one. How many shoes should he take out before he has a matching pair? (Assume that what is taken out is not put back.)

(a) 5 (b) 6 (c) 10 (d) 14

***185.** Gopal was given an apple and a knife. He was asked to make it into a cube. What is the minimum number of cuts that he needs to make?

(a) 4 (b) 6 (c) 8 (d) 12

***186.** Sankar asked Saleem to cut his chappathi into as many pieces as possible in 3 cuts. No piece can be moved until the third cut. Saleem did it right. How many pieces did he make?

(a) 5 (b) 6 (c) 7 (d) 8

***187.** Akbar asked Amar to cut his chappathi into as many pieces as possible in 4 cuts. No piece can be moved until the third cut. Amar did it right. How many pieces did he make?

(a) 8 (b) 9 (c) 10 (d) 11

***188.** A can is filled with 5 paise coins. Another can is filled with 10 paise coins. Another can is filled with 25 paise coins. All the cans are given wrong labels. If the can labeled *25 paise* is not having the 10 paise coins, what will the can, labeled *10 paise* have?

(a) 25 paise (b) 5 paise (c) 10 paise (d) Cannot be determined

***189.** A can is filled with 5 paise coins. Another can is filled with 10 paise coins. Another can is filled with 5 and 10 paise coins. All the cans are given wrong labels. You need to identify the can that has the 10 paise coins in it. You are allowed to inspect only one coin from a can, of your choice. Which can must you choose?

(a) The can that is filled with 5 paise coins.

(b) The can that is filled with 10 paise coins.

(c) The can that is filled with 5 and 10 paise coins.

(d) Cannot be determined

***190.** Sami wrote 5 different letters. He prepared 5 different envelopes for the 5 letters. If he randomly distributed the 5 letters to the 5 envelopes, what is the probability that each letter gets into the correct envelope?

(a) 1/2 (b) 1/5 (c) 1/60 (d) 1/120

***191.** Priya takes 6 minutes to walk to her school from her house. Bianca who lives in the same house can walk to the same school 8 times in an hour. Who walks faster?

(a) Priya (b) Bianca
(c) Cannot be determined from the facts given (d) None of these

***192.** Vinod took a certain number of tests. Out of 10, his scores on the first 9 tests were 1, 2, 3, 4, 5, 6, 7, 8, and 9. In all the other tests, he scored 10 out of 10. If his average score is 9, how many tests did he take?

(a) 40 (b) 50 (c) 60 (d) None of these

***193.** AB and XY are 2 two-digit numbers. A, B, X, and Y are assigned values from 5, 6, 8, and 9. How should the assignment be done so that AB – XY is minimum?

(a) A=9, B=5, X=8, and Y=6 (b) A=9, B=8, X=5, and Y=6
(c) A=9, B=5, X=6, and Y=8 (d) None of these

***194.** 10 machines can cut 100 papers in 10 minutes. How many minutes does it take 20 machines to cut 200 papers?

(a) 10 (b) 20 (c) 30 (d) 40

***195.** 10 machines can cut 100 papers in 10 minutes. How many papers will be cut by 5 machines in 1 hour?

(a) 200 (b) 300 (c) 400 (d) 500

***196.** Siva, Varma, and Patil ran a 100 meter race. Siva finished first beating Varma by 20 meters, and Patil by 30 meters. If Varma and Patil run a 100 meter race, with Varma giving Patil a head start of 10 meters, who will win the race?

(a) Siva (b) Varma
(c) Patil (d) Cannot be determined from the given facts.

***197.** A six-digit number 123ABC is exactly divisible by 5, 7, and 9. How many such possible numbers are there?

(a) 2 (b) 3 (c) 4 (d) 5

***198.** A train traveling at 60 km/h takes 3 seconds to enter a tunnel. The same train takes 30 seconds to completely come out of the tunnel. What is the length of the tunnel in meters?

(a) 400 (b) 500 (c) 600 (d) None of these

***199.** Here are the statements of 4 boys.

Mani : Subbu ate it

Subbu : Joshi ate it

Kumar : I didn't eat it

Joshi : I didn't eat it

Only one of them is telling the truth. Who ate it?

(a) Mani (b) Subbu (c) Kumar (d) Joshi

***200.** AB and BA are 2 two-digit numbers such that AB + BA = CAC. What is A+B+C? (Assume C is not 0)

(a) 13 (b) 14 (c) 15 (d) None of these

***201.** You are given a 3 liter can, a 5 liter can, and a bucket of water. You need to use only these two cans to get exactly 4 liters of water in the 5 liter can. Is it possible?

(a) Yes
(b) No
(c) No. But, possible if the 4 liters is to be in the bucket.
(d) None of these

***202.** A railway track passes through a tunnel. Raman and Gopal are inside the tunnel at a distance of two-fifth from one end, when they heard the sound of a train approaching the tunnel, Raman ran towards one end of the tunnel and Gopal ran towards the other immediately. They both ran at a speed of 15 mi/hr. But, both of them just managed to escape. The speed of the train in miles per hour is,

(a) 75
(b) 83
(c) 84
(d) cannot be determined from the given facts.

***203.** There are 3 bulbs inside a room. There are 3 switches outside the room. You can enter the room only once. Is it possible to find which switch controls which bulb?

(a) Yes
(b) No
(c) No, but possible if allowed to go into the room more than once
(d) None of these

***204.** Two people at the two ends of a road tunnel of length 150 km start at two bikes facing each other at 25 km/hr and 50 km/hr respectively. At the same moment, a bird starts flying from one end at 100 km/hr towards the other end until it meets the other person. Once it meets, it reverses direction, and starts flying towards the other person. The bird continues this pattern until the bikes collide head-on. What is the total distance traveled by the bird in kilometers?

(a) 100
(b) 200
(c) 300
(d) 400

***205.** What is the minimum number of standard weights that can measure any of 1, 2, 3, 4, 5, 6, 7, 8 kg?

(a) 2
(b) 3
(c) 4
(d) 5

***206.** There are two squares of sides 10 m and 30 m respectively. The smaller square is placed inside the larger one such that the centers coincide and the sides are parallel. The area outside the inner square, but inside the outer square is filled with water. What is the minimum number of square-shaped metal sheets that are needed to reach the inner square from the outer square? Assume the metal sheets can be welded together and the side of the metal sheet measures 1m.

(a) 7
(b) 8
(c) 9
(d) 10

207. How many squares do you see in this picture?

(a) 12
(b) 13
(c) 1
(d) 20

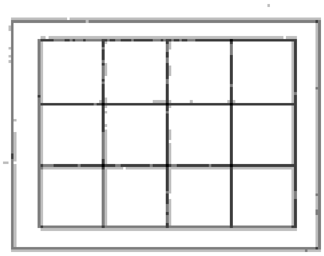


Fig. 4.2

***208.** A rice seller has a balance to measure any quantity of rice that could weigh between 1-40 kg as a whole number. The minimum number of standard weights needed is

(a) 4
(b) 5
(c) 6
(d) 7

***209.** The probability that it will rain today is 0.5. The probability that it will rain tomorrow is 0.6. The probability that it will rain either today or tomorrow is 0.7. What is the probability that it will rain today and tomorrow?

(a) 0.3 (b) 0.25 (c) 0.35 (d) 0.4

210. The determinant of the following matrix is

$$\begin{pmatrix} 6 & -8 & 1 & 1 \\ 0 & 2 & 4 & 6 \\ 0 & 0 & 4 & 8 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

(a) 11 (b) −48 (c) 0 (d) −24

211. Let $A = (a_{ij})$ be a n-rowed square matrix and I_{12} be the matrix obtained by interchanging the first and the second rows of the n-rowed identity matrix. Then AI_{12} is such that its first

(a) row is same as the second row.

(b) row is same as the second row of A.

(c) column is the same as the second column of A.

(d) row is all zero.

***212.** What is the maximum value of the function $f(x) = 2x^2 - 2x + 6$ in the interval [0, 2]?

(a) 6 (b) 10 (c) 12 (d) 5.5

***213.** Given $\sqrt{224)_r} = 13)_r$ the value of radix r is

(a) 10 (b) 8 (c) 5 (d) 6

***214.** The number of equivalence relations of the set {1,2,3,4} is

(a) 15 (b) 16 (c) 24 (d) 4

215. Which of the following propositions is a tautology?

(a) $(p \vee q) \rightarrow p$ (b) $p \vee (q \rightarrow p)$ (c) $p \vee (p \rightarrow q)$ (d) $p \rightarrow (p \rightarrow q)$

216. Let R be a reflexive and transitive relation defined on a set D. A new relation E is defined on set D such that

$$E = \{ (a,b) \mid (a,b) \in R \text{ and } (b,a) \in R \}$$

The relation E is

(a) a partial order (b) a total order

(c) an equivalence relation (d) none of the above

217. Let R be a reflexive and transitive relation defined on a set D. A new relation E is defined on set D such that

$$E = \{ (a,b) \mid (a,b) \in R \text{ and } (b,a) \in R \}$$

A relation ≤ is defined on the equivalent classes of E such that $E_1 \leq E_2$ if there exists a,b such that $a \in E_1$, $b \in E_2$ and $(a,b) \in R$. This relation is,

(a) a partial order (b) a total order

(c) an equivalence relation (d) none of the above

218. A, B are two 8-bit numbers such that $A+B \leq 2^8$. The number of possible combinations of A and B is

(a) 2^9 (b) 2^8 (c) 2^{16} (d) $2^4 - 1$

Answers

1. c	2. d	3. b	4. a	5. c
6. d	7. a	8. b	9. c	10. a
11. a, b	12. d	13. c	14. d	15. c
16. c	17. d	18. b	19. d	20. c
21. d	22. a	23. d	24. c	25. a
26. d	27. b	28. d	29. c	30. a
31. a	32. b	33. c	34. d	35. a
36. c	37. c	38. b	39. a, b, c	40. c
41. a	42. c	43. d	44. a	45. d
46. b	47. c	48. c	49. c	50. c
51. a	52. d	53. a	54. a	55. a
56. c	57. a	58. c	59. a	60. a, b
61. a	62. b	63. c	64. a	65. b
66. c	67. a	68. c	69. b	70. a
71. d	72. a	73. c	74. d	75. c
76. c	77. a, b, d	78. a, d	79. a, b, d	80. d
81. d	82. b, c	83. c	84. c	85. a, b, c
86. a, b, c	87. a	88. a, b	89. c	90. a, d
91. c, d	92. a, b, c, d	93. c	94. a, d	95. b
96. a	97. c	98. b	99. c	100. b, c, d
101. b, d	102. c	103. b	104. c	105. b, d
106. a, d	107. a, b	108. c	109. b, c	110. a, b
111. a, b	112. c	113. d	114. a, b, c, d	115. a
116. c	117. c	118. c	119. c	120. d
121. a	122. d	123. d	124. a, b, c	125. a
126. d	127. a	128. d	129. c	130. c
131. c	132. d	133. b, d	134. a	135. d
136. c	137. b	138. a	139. a	140. d
141. d	142. d	143. d	144. b	145. b
146. a	147. c	148. c	149. c	150. b
151. d	152. a	153. b	154. b	155. c
156. c	157. d	158. c	159. d	160. b
161. d	162. a	163. b	164. d	165. b
166. c	167. c	168. c	169. a	170. d
171. d	172. a	173. a	174. d	175. d
176. c	177. c	178. c	179. c	180. d

181. c	182. a	183. d	184. c	185. b
186. c	187. d	188. a	189. c	190. d
191. a	192. d	193. a	194. a	195. b
196. b	197. b	198. d	199. c	200. d
201. a	202. a	203. a	204. b	205. b
206. b	207. d	208. a	209. d	210. b
211. c	212. b	213. c	214. a	215. c
216. c	217. a	218. a		

Explanations

1. ${}^{39}P_{30} / {}^{40}P_{30} = 1/4$

2. Probability that the unit digit is not 7 is 9/10.
Probability that the tens digit is not 7 is 9/10.
Probability that the hundreds digit is not 7 is 8/9.
So, the probability that all the three digits are not 7 is (9/10) (9/10) (8/9) = 18/25.

3. Let $x = 0.15252525\ldots$
$1000\,x - 10\,x = 151$. So, $x = 151/990$

4. The required value is, number of arrangement without restriction – number of arrangement with restriction. That is
$(8-1)! - (7-1)!\,2! = 6!\,(7-2) = 3600$

5. The converse is also true.

6. Total cases is $6 \times 6 \times 6 = 216$
For real roots $b^2 \geq 4\,ac$
When $b^2 = 36$, ac can't be 10, 11, 12...
When ac is 1, b can take the 5 values 2, 3, 4, 5, 6.
When ac is 2, either $a = 1, b = 2$ or $a = 2, b = 1$. When $ac = 2$, b can take the 4 values 3, 4, 5, 6. So, total 4 + 4 = 8 possible values. Continuing this way, we find there are 43 possible cases. Hence the required probability is 43/216.

7. Including 0 occupying the most significant position, the sum will be
$24\,(2 + 4 + 6 + 8)\,(10000 + 1000 + 100 + 10 + 1) = 24 \times 20 \times 11111$ ...I
Out of these, 0 occupies the most significant place in 4! numbers. Sum of these will be
$(4!/4)(2 + 4 + 6 + 8)\,(1000 + 100 + 10 + 1) = 6 \times 20 \times 1111$ ...II
I – II gives the result.

8. It is ${}^6C_2 \times {}^4C_2 / 2 = 45$

9. O(h) is 2, implies hh = e (e is the identity element of the group).
Now, $(ghg^{-1})(ghg^{-1}) = gh(g^{-1}g)hg^{-1} = g(hh)g^{-1} = gg^{-1} = e$.
So, $O(ghg^{-1})$ is 2.

13. $\log \sin(x) > 0 \Rightarrow \sin x > e^0 = 1$, which is impossible.

14. Only when $\lambda_{,} = 4$ and $\mu = 2$, we have 2 equations in three variables, giving infinitely many solutions.

15. Let *aRb*. Since the relation is symmetric, *bRa*. Since transitivity holds good, *aRb* and *bRa* imply *bRb* and *aRa*. If *R* has to be an equivalent relation, it has to be reflexive, ie., for any *x* belonging to *A*, *xRx* should be valid. Hence *R* need not be reflexive. So it need not be an equivalent relation. For example, let $A = \{1, 2, 3\}$. Let $R = \{(1, 2), (2, 1), (1, 1), (2, 2)\}$. *R* is both symmetric and transitive but not reflexive as (3, 3) is missing.

16. The set has 3 elements. So the power set has $2^3 = 8$ elements.

17. It is 9.

18. It is $\sqrt{18^2 + 6^2 + (4.5)^2}$

19. Out of the 8 available squares, 6 can be selected in ${}^8C_6 = 28$ ways. This includes the two possibilities which are not allowed. These two possibilities are—One with top row empty and the other with the bottom row empty. So, there are $28 - 2 = 26$ possibilities.

20. The required number is $100 - (10 + 20 + 15 + 10 + 12 + 5 + 8) = 20$.

26. Solving the three equations we get $n = 9$ and $r = 3$.

27. Consider the function $f(x) = x - \cos(x)$. $f(0)$ is $0 - 1 = -1$, a negative number. $f(\pi)$ is $\pi - (-1) = \pi + 1$, a positive number. So, the function will have an odd number of roots (Ref Qn. 54) in the interval $[0, \pi]$. Also, it cannot have infinitely many roots as $\cos(x)$ oscillates in $[-1, 1]$, while $(y =)\ x$ increases monotonically and so infinite solution is impossible. Hence the answer is option b.

28. It is $10^5 - {}^{10}P_5$

29. ${}^nC_r + {}^nC_r = {}^{(n+1)}C_r$

So, ${}^{47}C_4 + {}^{47}C_3 = {}^{48}C_4$, etc., Using this, the given summation can be simplified to ${}^{52}C_4$

30. The rank of a matrix is said to be *N*, if the determinant value of at least 1 sub-matrix of order $N \times N$ is not 0 and all $(N + 1) \times (N + 1)$ is zero. So the rank of the given matrix is 1, as any sub-matrix of order 2×2 has 0 determinant value and (1), a sub-matrix of order 1×1 has the non-zero determinant value of 1.

32. The number of elements in the power set of a set with *n* elements is 2^n. The given set *A* has *n* elements. So, $A \times A$ will have n^2 elements. So, its power set will have 2^{n^2} elements.

33. It is $1 - (0.6)(0.6)(0.6) = 0.784$.

34. Let $f(x) = x + 1/x$.

$f'(x) = 1 - 1/x^2$

$f'(x) = 0 \Rightarrow x = \pm 1$

$f''(x) = 2/x^3 > 0$, for $x > 0$. So, $x = 1$. $f'(1) = 2$

35. $f''(0) = 0$, as *f* is continuous at $x = 0$.

$$\lim_{h \to 0} f(x+h) = \lim_{h \to 0} f(x) + f(h) \Rightarrow \lim_{h \to 0} f(x) + \lim_{h \to 0} f(h) = f(x)$$

So, *f* is continuous for all *x*.

36. $f'(5) = \lim_{h \to 0} \frac{f(5+h) - f(5)}{h} = f(5) \lim_{h \to 0} \frac{f(h) - f(0)}{h} = f(5)\, f'(0) = 6$

38. Distinct elements of $\bigcup_{i=1}^{30} A_i = 30 \times 5/10 = 15$

Distinct elements of $\bigcup_{j=1}^{N} B_j = (n \times 3)/9 = n/3$

So, $n/3 = 15$ or $n = 45$

40. Given that 1, ω, ω^2 are the roots of $x^3 - 1 = 0$. So, $1 + \omega + \omega^2 = 0$; $1 \times \omega \times \omega^2 = \omega^3 = 1$. The given equation being a third degree equation has 3 roots. Let it be a, b, c. We have $a + b + c = -(-3)$ and $ab + bc + ac = 3$ and $abc = -7$. Only option c satisfies all these. Verify.

41. Consider the function $\phi(x) = f(x) - 2g(x)$

$\phi(0) = \phi(1) = 2$. So, $\phi(x)$ satisfies the conditions of Roll's theorem in [0, 1]. So, $\phi'(x) = f'(x) - 2g'(x)$ has at least one 0 at C in (0, 1)

i.e., $\phi'(C) = 0 \Rightarrow f'(C) = 2g'(C)$

42. The only possibility is $f(z) \neq 2$ is true and the other two are false. So, $f^{-1}(1) = y$

43. $h'(x) = 2f(x)\, f'(x) + 2g(x)\, g'(x)$.

So, $h'(x) = 2f(x)g(x) + 2g(x)(-f(x)) = 0 \Rightarrow h(x)$ is constant. So, $h(5) = 11$

45. Let $x = i^i$ Taking log on both sides, we get $\log x = i \log(i)$.

$e^{i\theta} = \cos(\theta) + i \sin(\theta)$. Putting $\theta = \pi/2$, we get $e^{i\pi/2} = i$.

So, $\log x = i \log(e^{i\pi/2}) = i \times i\pi/2$, $\log(x) = -\pi/2$.

Hence $x = e^{-\pi/2}$ — a real number

47. The given decimal number can be written as

$(1 + 2) \times 2^{12} + (1 + 2 + 4 + 8) \times 2^8 + (1 + 4) \times 2^4 + (1 + 2)$

$= 2^{13} + 2^{12} + 2^{11} + 2^{10} + 2^9 + 2^8 + 2^6 + 2^4 + 2^1 + 2^0$. This has 10 one's.

49. We can arrange 1 or 2 or 3 or … r things at a time.

Number of ways of arranging 1 at a time is n

Number of ways of arranging 2 at a time is n^2

Number of ways of arranging 3 at a time is n^3 etc.,

So, total number of possible permutations is:

$n + n^2 + n^3 + \ldots + n^r = n(n^r - 1)/(n - 1)$

51. Apply change of base rule.

52. In an equation with rational coefficients, irrational roots and complex roots occur in pairs. So, $\sqrt{5} + \sqrt{7} + i$ as a root implies $-\sqrt{5} + \sqrt{7} + i$, $-\sqrt{5} - \sqrt{7} + i$, $\sqrt{5} - \sqrt{7} + i$, $-\sqrt{5} - \sqrt{7} - i$, $-\sqrt{5} - \sqrt{7} - i$, $\sqrt{5} - \sqrt{7} - i$, $\sqrt{5} + \sqrt{7} - i$, are also roots. An equation of degree n has exactly n roots (with possible repetition of some roots). Hence the answer.

53. By Descarte's rule of signs, the number of positive real roots can't be more than the number of changes of sign in $f(x)$, in this case 3, as there are 3 change of signs in $f(x)$. The number of negative roots can't be more than the number of change of signs in $f(-x)$, i.e., 2 in this case. So, altogether it can't have more than 5 real roots (obviously 0 is not a root). But this being a polynomial of degree 7, must have 7 roots. So, at least 2 imaginary roots must be present.

54. If you draw the graph of $f(x)$ between $x = a$ and $x = b$, it should cut the x-axis either 0 or even number of times. It will have as many roots as the number of times it is cutting the x-axis.

55. Ref Qn. 53. It cannot have more than 3 real roots. In an equation with real coefficients, imaginary roots occur in pairs. Here $f(0)$ and $f(\infty)$ are negative. So in the interval $(0, \infty)$ there can be none or even number of roots. Since $f(-1)$ is positive, it should have at least one root in $(-1, 0)$. Since $f(0)$ is negative and $f(\infty)$ is positive, it has at least 2 real roots, and not more than three. So, it has to be 3 because if it is 2, the number of imaginary roots will be 3, which is infeasible (since imaginary roots occur in pairs).

56. $f(-\infty)$ is positive and $f(+\infty)$ is also positive. But $f(0)$ is negative. So, it should have at least 1 root between $(-\infty, 0)$ and at least one root between $(0, +\infty)$.

57. The remainder on division will be a first degree polynomial. Let it be $Mx + N$. So $f(x) = (x - \alpha)(x - \beta)\, Q + (Mx + N)$. ($Q$ is the quotient). Putting $x = \alpha$, $f(\alpha) = M\alpha + N$ and $f(\beta) = M\beta + N$. Solving we get the answer.

58. If the base is greater than 1, it will be $-\infty$. If the base is less than 1, it will be $+\infty$. If the base is 1, it will be undefined.

59. We have $x^n + nax - b = (x - a_1)(x - a_2) \ldots (x - a_n)$

Differentiating both sides with respect to x and putting $x = a_1$, we get the result.

61. $|a - b| < n$ implies $-n < a - b < n$

$|b - c| < m$ implies $-m < b - c < m$

Adding both these inequalities,

$-(n + m) < a - c < n + m$, which is nothing but $|a - c| < n + m$.

63. A may win in the first or second …or nth toss. So, the required probability is $(1/2) + (1/2)^3 + (1/2)^5 + \ldots$ Summation of this geometric progression is 2/3.

64. Any number can be expressed as the product of prime numbers in a unique way. So, 200! written in this form will have a certain number of 2's and 5's. The number of 2's will be more than the number of 5's as each even number contributes at least one 2. The only way to get a 0 is to multiply a 2 by 5. The number of 5's will decide the number of zeroes. The numbers 5, 10, 15, … 200 each contribute one 5. This totals to 40. The numbers 25, 50, 75, …, 200 will contribute one more 5. The number 125 will contribute yet another. So, totally 40 + 8 + 1 = 49 zeroes.

65. Consider a $n \times n$ matrix. To find the determinant, we have to multiply each element of the first row, with its cofactor. The cofactor is the determinant value of a $(n - 1) \times (n - 1)$ matrix. The number of terms in the determinant value of a $n \times n$ matrix, $T(n) = n\, T(n-1) = n \times (n-1)\, T(n-2) \ldots = n!$ Here $n!$ is given as 720. So, n is 6.

67. It is defined if $|x| - x > 0$, i.e., $|x| > x$.

If $x \geq 0$, $|x| = x$. So, $x > x$, which has no feasible solution.

If $x < 0$, $|x| = -x$. So, $-x > x$, which has the solution $x < 0$.

68. The probability that the first ball drawn is white and the second black is $(10/25) \times (15/24) = 1/4$.

The probability that the first ball drawn is black and the second white is (15/25) × (10/24) = 14. So, the required probability is 1/4 + 1/4 = 1/2.

69. The iterative formula is $x_{k+1} = x_k + f(x_k) / f'(x_k)$. Here $x = \sqrt{b}$, i.e., $x^2 - b = 0$. Taking $f(x) = x^2 - b$, we get the answer.

72. The required value of x is, $x = -\sqrt{20+x}$. Solving, we get $x = -4$ or 5.

73. $P(\bar{A} \cap \bar{B}) = 1 - P(A \cup B) = 1 - (P(A) + P(B) + P(A \cap B)) = 0.39$.

74. By Lagrange's mean value theorem, in the interval [0,5], there must exist a constant C in (0, 5) such that $g'(C) = (g(5) - g(0)) / (5-0) = -5/6$

76. The first element of A may be mapped to any one of the n elements of B. The second element to any one of the remaining $n - 1$ elements. Proceeding this way, the m^{th} element can be mapped to one of the remaining $(n - m + 1)$ elements of B. So, we have $n \times (n-1) \times (n-2) \times \ldots (n - m + 1) = {}^nP_m$ possible ways.

77. In a set, order of the members and repetition is immaterial.

80. It is not reflexive as $-3\ R\ -3$ is not true. It is not irreflexive as $2\ R\ 2$ is true. It is not symmetric as $-3\ R\ 3$, but $3\ R\ -3$ is not true. It is anti-symmetric as, if $a\ Rb$ and $b\ Ra$ are both true, then $a = b$.

81. We have $1\ R\ 2$ and $2\ R\ 1$, but $1\ R\ 1$ is not true. So, R is not transitive. We have $4\ R\ 3$, but $3\ R\ 4$ is not true. So, R is not symmetric. Also, $1\ R\ 2$ and $2\ R\ 1$, but $1 \neq 2$. So, R is not anti-symmetric.

82. A relation is a function if and only if each element in the domain has a unique image. (a) is not a function as the element 1 has two images 2 and 3. (d) is not a function as the element 3 in the domain has no image.

86. All three can be proved by Pigeon-hole principle.

93. The roots are given by $x = \left(-b \pm \sqrt{b^2 - 4\,ac}\right)/2a$

Differentiating both sides, treating c as a variable, $dx = dc / \sqrt{b^2 - 4\,ac}$

For dx to be less than 5×10^{-5}, dc should be to the accuracy of approximately 8×10^{-5} (check by putting $b = -2$, $a = 1$ and $c = \log 2$, and solving the above equation).

94. The determinant of a matrix equals the product of its eigenvalues.

95. M has n rows. If all the elements of a row are multiplied by 2, the determinant value becomes 2×5. Multiplying all the n rows by 2, will make the determinant value $2^n \times 5 = 40$. Solving, $n = 3$.

98. To ensure 5 digit accuracy, the error term $x^{2n+2}/(2n+2)!$ should be less than 5×10^{-6} Solving, we get $n = 7$.

102. All are parallel lines, but only the second line has chances of bounding the convex region.

119. Substitute and verify. The solution can also be obtained like this—The complementary equation is $D^2 + 3\,D + 2 = 0$. This has $D = -1, -2$ as the roots. Hence the solution is: $C_1e^{-x} + C_2e^{-2x}$.

120. By definition $\neg P \Rightarrow Q$ is true means that $\neg(\neg P)\ V\ Q$ is true. The proposition $\neg P\ V\ (P \Rightarrow Q)$ is nothing but $\neg P\ V\ (\neg P\ V\ Q)$, i.e., $\neg P\ V\ Q$. The trueness or the falsity of this cannot be determined from the given proposition.

121. $600 = 5 \times 5 \times 3 \times 2 \times 2 \times 2$. Any factor of 600 can be obtained by choosing 1, 2, 3 or not two's at all, i.e., 4 ways of selecting two. Similarly, there are 2 ways of choosing 3, and 3 ways of choosing 5. So, there are altogether $4 \times 2 \times 3 = 24$ different ways. This includes 1 (corresponding to choosing no two's, no three's and no five's) as well as 600 (by choosing all the 2's, 3's and 5's).

122. It can be written as $\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 4+1 & 5+2 & 6+3 \end{pmatrix}$

So, $\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 5 & 2 & 9 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 1 & 7 & 3 \end{vmatrix} + \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 4 & 5 & 6 \end{vmatrix} = 0 + 0 = 0$

123. None of the elementary operations affects the rank.

125. The order of a subgroup should divide the order of the group. 11 being a prime number, number has no proper divisor and hence it can't have any proper sub-group.

126. We have $(AB)' = B'A'$. Since A and B are symmetric matrices, $A' = A$ and $B' = B$. Hence the answer.

128. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

i.e., $P(A \cup B) + P(A \cap B) = P(A) + P(B)$

Since $P(A \cap B)$ cannot be negative, $P(A \cup B) \leq P(A) + P(B)$

130. $(x*y)^2 = (x*y)\ (x*y) = (x*y)\ (y*x) = x*\ (y*y)\ \ *x = x*y^2*x$

$= x*x*\ y^2 = x^2*y^2$.

132. Number of strings of length 1 is n.

Number of strings of length 2 is $n - 1$.

Number of strings of length 3 is $n - 2$, etc.

Number of strings of length $n - 1$ is 2.

Number of strings of length n is 1.

Totally, $1 + 2 + 3 \ldots + n = n(n+1) / 2$

134. We have $2R5$ and $5R2$. If it is transitive, then $2R2$, but it is not, as $2R2$ means 2 is not equal to 2, which is wrong.

136. $A = (A \cap B) \cup (A \cap \bar{B})$.

$P(A) = P(A \cap B) + P(A \cap \bar{B})$

$= Q + R$

142. 1 is the identity element as 1 LCM $a = a$ LCM $1 = a$, for any a belonging to the set. What is the inverse of 3? If it is y, then 3 LCM $y = y$ LCM $3 = 1$. No such y can be found.

143. 24 is the identity element, as 24 GCD $x = x$ GCD $24 = x$, for any x belonging to the set. But inverse doesn't exist.

145. The total number of possible functions is n^m. If it is to be 10, then $n = 10$ and $m = 1$.

147. $g(-x) = f(-x)[f(-x) + f(x)]$. If f is even then $f(-x) = f(x)$.
So, $g(-x) = f(x)[f(x) + f(-x)] = g(x)$. Hence g is even if f is even.

150. ω is a generator, as all the cube roots of unity can be expressed as powers of ω.
For similar reasons, ω^2 is also a generator, as $\omega = (\omega^2)^2$ and $1 = (\omega^2)^3$.

151. $x \log x = \log x / (1/x)$. Apply L'Hospital's rule.

152. Whenever $f(a)$ and $f(b)$ are of different signs, then f has odd number of roots (at least one) between a and b.

153. Let $S = x + 2x^2 + 3x^3 + \ldots$

$$xS = x^2 + 2x^3 + \ldots$$

$$S - xS = x + x^2 + x^3 + \ldots = x(1 - x^n) / (1 - x) = x / (1 - x)$$

$$(1 - x)S = x / (1 - x). \text{ So, } S = x / (1 - x)^2.$$

Another way of doing this is:

$$S / x = \Sigma kx^{k-1} = \mathrm{d}/\mathrm{d}x\ \Sigma x^k = \mathrm{d}/\mathrm{d}x\ [\ x/(1 - x)] = 1/(1 - x)^2$$

So $S = x / (1 - x)^2$

154. A linear transformation F satisfies $F\ (mx, my) = mF\ (x, y)$ and
$F\ (a + b, c + d) = F\ (a, c) + F\ (b, d)$. Option ($b$) does not satisfy this while the others do.

155. Eigenvalue m satisfies the equation $|A - mI| = 0$. Put $m = 0$. We get $|A| = 0$, which is given to be true. So, $m = 0$ is an eigenvalue.

165. Each print is a k-tuple $I1, I2\ldots, Ik$ — such that
$N \geq I1 \geq I2 \geq \ldots \geq I_k \geq 1$.
Hence the problem reduces to choosing k integers, with repetitions allowed, from 1, 2, 3…, N — which is ${}^{k+N-1}C_k$. This is because any such selection, if written in ascending order will satisfy the conditions and any solution will be a selection.

170. Square of an odd number is an odd number. Square of an even number is an even number. If you add two odd numbers, you get an even number. As a result, (odd number)^2 + (odd number)^2 = even number. Since no even number can be a square of an odd number, options a, b, and c, cannot be correct.

171. Square of an odd number is an odd number. Square of an even number is an even number. If you add two even numbers, you get an even number. As a result, (even number)^2 + (even number)^2 = even number. Since no even number can be a square of an odd number, options a, b, and c, cannot be correct.

172. You can verify by constructing truth table for an implication, say, $P \to Q$ and its contrapositive $\neg Q \to \neg P$

174. Let us prime factorize the number. 637245 is $5 \times 3 \times 3 \times 7 \times 7 \times 17 \times 17$. The word we are looking for must have the letter corresponding to 17, which is Q.

175. Let us prime factorize the number. $124950 = 5 \times 5 \times 2 \times 3 \times 7 \times 7 \times 17$. The word we are looking for must have the letter corresponding to 17, which is Q. It is not a bad idea to guess Q will be immediately followed by a U. The code for U is 21. We are left with $5 \times 5 \times 2 \times 7$. If the remaining letters are 4, it has to be E, E, B, G. But there is no 6-letter word with the letters Q, U, E, E, B, G. So, let us assume that there are only 3 remaining letters. The possible codes for these 3 letters are – (25,2,7), (10,5,7), (5,5,14). This means the possible letters are – (Y, B, G), (J, E, G), (E, E, N). Remember Q and U are the other 2 letters. It is not difficult to find (E, E, N) is the correct one and the word is QUEEN.

176. Let us prime factorize the number. 3135 = 3 × 5 × 11 × 19. The alphabet with the code 19 must be present in the word we are looking for. It is S. The alphabet with the code 11 must also be present in the word we are looking for. It is K. We are left with the factors – 3 and 5. They may account for the single alphabet O (this has the code 15) or they may account for the alphabets – C and E. Let us pursue our search with the alphabet O. We are looking for a word made up of the letters – S, K, O. No such word exists. Since 3125 is same as 3125 × 1, the alphabet with code 1 can be used. So the alphabet A can also be used. So, we are looking for a word made up of S, K, O, A. The word is SOAK. If 3 and 5 account for the alphabets – C and E, we are looking for a word made up of S, K, C, E. Since the alphabet A can also be used, the word is CAKES.

177. Let us prime factorize the number. 1265 = 5 × 11 × 23. The alphabet with the code 23 must be present in the word we are looking for. It is W. The alphabet with the code 11 must also be present in the word we are looking for. It is K. We are left with the factor 5. It represents the alphabet E. We are looking for a word made up of the letters—W, K, E. No such word exists. Including the alphabet A, we are looking for a word made up of the letters—W, K, E and A. It could be WEAK or WAKE.

178. The largest 10-digit number is 9876543210, which is not the correct answer as it is not divisible by 4. Note that a number is divisible by 4 if the last two digits are divisible by 4. The number we are looking for cannot be got by swapping the 1 and 0. Let us try permuting the last 3 digits—2, 1, and 0. Since we are for the largest number, we need to try in the order—120, 102, 021, 012. Since 120 is divisible by 4, the correct answer is 9876543120.

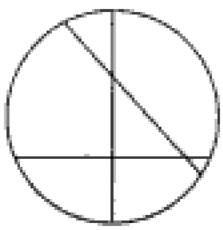
179. The largest 10-digit number is 9876543210, which is not the correct answer as it is not divisible by 8. Note that a number is divisible by 8 if the last three digits are divisible by 8. The number we are looking for cannot be got by swapping the 1 and 0. Let us try permuting the last 3 digits—2, 1, and 0. Since we are looking for the largest number, we need to try in the order—120, 102, 021, 012. Since 120 is divisible by 8, the correct answer is 9876543120.

180. The smallest 10-digit positive number is 1023456789, which is not the correct answer as it is not divisible by 8. Note that a number is divisible by 8 if the last three digits are divisible by 8. The number we are looking for cannot be got by swapping the 1 and 0. Let us try permuting the last 3 digits—7, 8, and 9. Since we are for the smallest number, we need to try in the order—789, 798, 879, 897, 978, 987. none of these is divisible by 8. Let us enlarge our search domain by permuting the last 4 digits—6, 7, 8, and 9. We need to try in the order—6789, 6798, 6879, 6897, 6978, 6987, 7689, 7698, 7869, 7896, 7968, 7986 etc., The first number in this order that is divisible by 8 is 7896. So, the number we are looking for is 1023457896.

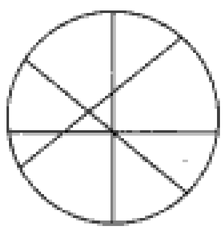
181. A number is divisible by 11 if the difference of the sum of the numerals in the odd numbered positions and the sum of the numerals in the even numbered positions is divisible by 11. Consider the largest number—9876543210. The sum of the numerals in the odd numbered positions is 25 (9+7+5+3+1). The sum of the numerals in the even numbered positions is 20 (8+6+4+2+0). The difference is 5, which is not divisible by 11. So, 9876543210 is not divisible by 11. We have to permute the numerals so that the sum of the numerals in the odd numbered positions becomes 28. This is because the sum of the numerals in the even numbered positions will then become 17, making the difference 11, which is divisible by 11. This can be achieved by swapping 4 and 1. The number is 9876513240. This is divisible by 11. The numerals in the odd numbered positions are 9, 7, 5, 3, 4. Arranging them descending order (because we are in the look out for the largest number), we get 9, 7, 5, 4, 3, as the correct order of the numerals in the odd numbered positions. Doing the same with numerals in the even numbered positions, the correct order of the numerals in the even numbered positions will be 8, 6, 2, 1, 0. Therefore the largest 10-digit integer, containing all the numerals 1,2,3,4,5,6,7,8,9,0 that is divisible by 11 is 9876524130.

185. Each cut should give him a face of the cube. A cube has 6 faces.

186. To get as many pieces as possible, each line must cut all the existing lines at the non-intersecting points.



187. To get as many pieces as possible, each line must cut all the existing lines at non-intersecting points.



188. A diagram will make it easy to comprehend.

It is easy to find that the can labeled 25 Paise must have 5 paise coins in it. So, the can labeled 10 Paise, must have 25 paise coins in it.

189. The can labeled 5/10 paise.

A diagram will make it easy to comprehend.

The can labeled 5/10 Paise will have either all 5 paise or all 10 paise. If the coin you inspected is a 5-paise coin, the can labeled 5 Paise must have the 10 Paise coins in it. If the coin you inspected is a 10 Paise coin, that is the can you are looking for.

190. There are 5! ways of distributing the 5 letters to the 5 envelopes. Out of these, there is only one way that correctly distributes the letters to the envelopes.

191. In 1 hour, Priya can walk to her school 10 times.

192. Let m be the number of tests he took. His average is $\frac{45 + (m - 9) \times 10}{m}$

Equating the average to 9 and solving for m, we get m = 45.

193. A = 6, B = 8, X = 5, and Y = 9 is another possible answer.

194. 1 machine can cut 10 papers in 10 minutes.

1 machine can cut 1 paper in 1 minute.

20 machines can cut 20 papers in 1 minute.

20 machines can cut 200 papers in 10 minutes.

Another way to reason out is to consider the 20 machines as two groups of 10 machines each. Each group can cut 100 papers in 10 minutes. So, together they can cut 200 papers in 10 minutes. So, the statement—10 machines can cut 100 papers in 10 minutes, expressed algebraically is, 100 machine-minutes is equivalent to 100 papers. The question expressed algebraically is, finding the number of minutes (let us call it) m, such that 20 × m = 200.

195. 1 machine can cut 10 papers in 10 minutes.

1 machine in 1 minute can cut 1 paper.

5 machines in 1 minute can cut 5 papers.

5 machines in 60 minutes can cut 300 papers.

Another way to reason out is to understand the fact that what could be done by 5 machines in 1 hour is essentially same what could be done by 10 machines in 30 minutes.

So, the statement—10 machines can cut 100 papers in 10 minutes, expressed algebraically is, 100 machine-minutes is equivalent to 100 papers. The question expressed algebraically is, what is the equivalent of 300 machine-minutes (the 300 is 5 machines × 60 minutes).

196. It is obvious that Varma runs faster than Patil. By the time Varma finished running 80 meters, Patil could run only 70 meters. By giving a head start of 10 meters, they will be tied when they are 20 meters to the finish line. Since Varma runs faster than Patil, he will cover the remaining 20 meters before Patil.

197. The least common multiple of 5, 7, and 9 is 315. If the six-digit number 123ABC is exactly divisible by 5, 7, and 9, it has to be a multiple of 315. The number 123ABC can be written as 123000 + ABC. Dividing 123000 by 315 leaves the remainder 150. So, ABC must leave a remainder of 165 when divided by 315. This gives us the number 123165. Adding 315 or any multiple of it still gives us a number exactly divisible by 5, 7, and 9. So, there are 3 possible numbers – 123165, 123480 (123165+315), and 123795 (123165+315+315).

198. Let the length of the train be A.

A = 1/20 km.

Let the length of the tunnel be C.

A+C = 1/2 km.

So, the length of the tunnel is 9/20 km. i.e., 450 meters.

199. First assume only Mani's statement is true. The story with this assumption reads ...
Subbu ate it. Joshi didn't eat it. Kumar ate it...
This is a contradiction. So Mani's statement is not true.
Assume only Subbu's statement is true. The story with this assumption reads ...
Subbu didn't eat it. Joshi ate it. Kumar ate it...
This is also a contradiction. So Subbu's statement is also not true.
Assume only Kumar's statement is true. The story with this assumption reads ...
Subbu didn't eat it. Joshi didn't eat it. Kumar didn't eat it. Joshi ate it.
This is also a contradiction. So Joshi's statement is also not true.
Assume only Joshi's statement is true. The story with this assumption reads ...
Subbu didn't eat it. Joshi didn't eat it. Kumar ate it. Joshi didn't eat it.
This is not a contradiction. So Joshi's statement is the true statement. Accordingly, Kumar ate it.

200. Addition of two 2-digit numbers cannot be greater than 198. So, C is 1.
AB can be written as 10A + B.
So, AB + BA = 1A1 can be written as (10A + B) + (10B+A) = 100 + 10A + 1
Simplifying,
11B = 101 − A
Since 11B is a multiple of 11, 101-A has to be a multiple of 11. This can happen only if A is 2. So, C=1, A=2, B=9. Hence A + B + C is 12.

201. Fill the 5 liter can.
Pour it into the 3 liter can.
The 5 liter can will be left with 2 liters of water.
Empty the 3 liter can.
Transfer the 2 liters in the 5 liter can to the 3 liter can.
The 5 liter can is now empty.
Fill it from the bucket.
Fill the 3 liter can (which is having 2 liter now) from the 5 liter can.
What is left in the 5 liter can will be 4 liters.

202. No generality is lost by assuming, Raman ran towards the closer end of the tunnel, say A, and Gopal ran towards the farther end of the tunnel, say B. Let the length of the tunnel be y miles. Time taken by Raman to reach the end of the tunnel is $2y/75$ hours (since distance traveled is $2y/5$ and the speed is 15 miles/hour). Time taken by Gopal to reach the far end of the tunnel is $3y/75$ hours (since distance traveled is $3y/5$ and the speed is 15 miles/hour).

The train entered the tunnel when Raman just reached it and the train was at the other end of the tunnel when Gopal just reached it. This means the train took $3y/75 - 2y/75 = y/75$ hours to cover the tunnel.

Speed of the train = Distance covered / Time taken to cover
Distance covered is y miles.
Time taken is $y/75$.
Therefore, the speed of the train is 75 miles/hour.

203. Let us call the bulbs A, B, and C. Switch on one of them, say A. Wait for a couple of minutes and switch it off. Now, switch on another bulb, say B, and enter the room. The bulb that is glowing corresponds to the switch B. Touch the other 2 bulbs. The one that is warmer corresponds to the switch A. The left out bulb corresponds to the switch C.

204. Find the time taken for the bikes to collide. It is 2 hours. Distance traveled by the bird in 2 hours is, 200km.

205. The standard weights are 1kg, 3kg, and 4kg.

206. The shortest distance between the two squares is 10m. To cover this distance using the minimum number of metal sheets, the sheets have to welded diagonally. The diagonal measures $\sqrt{2}$. So, we need $10/\sqrt{2}$ sheets. So, we need 8 sheets.

208. The weights are 1kg, 3kg, 9kg, and 27kg. With these weights he can weigh anything between 1 – 40kg as a whole number.

209. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cap B) = 0.5 + 0.6 - 0.7 = 0.4$

212. $f'(x) = 4x - 2$. $f(x)$ increases if $f'(x) > 0$. i.e., $f(x)$ increases if $x > ½$. So, maximum value is attained at $x = 2$.

213. $\sqrt{2r^2 + 2r + 4} = r + 3$. Solving, r = 5 or –1. Since negative radix is illogical, r = 5.

214. {(1,1), (2,2), (3,3), (4,4)} is an equivalence relation and part (subset) of any other set that is an equivalence relation, as any equivalence relation has to be reflexive.

We cannot construct another equivalence relation by adding a single ordered pair as symmetric property should hold good. Let us find how many equivalence relations can be got by adding 2 ordered pairs. If we add (1,2), we need to add (2,1), to satisfy the symmetric property. Since there are 4 elements, 2 elements can be selected in ${}^4C_2 = 6$ ways. Similarly, some equivalence relations can be got by adding 4 ordered pairs, like adding (1,2), (2,1), (3,4), (4,3). Possible cases covered under these category is ${}^4C_2 / 2 = 3$ ways. Likewise the number of possible equivalence relations that can be got by adding 6 ordered pairs, like (1,2), (2,3), (3,1), (2,1), (3,2), (1,3), will be ${}^4C_3 = 4$. There is only one possible equivalence relation by adding 8 ordered pairs. So, totally 15 possible equivalence relations can be got.